Consensus based distributed estimation of biomass concentration in reverse osmosis membranes

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Introduction

- **Biofilm**: bacteria accumulation over a certain surface (substratum).
- Biofilm formation is a severe problem in desalination plants.
- **Reverse osmosis** (RO) become less permeable.
- Traditional pretreatments and biocide dosing **not effective** after irreversible attachment phase.
- Early *detection* and *estimation* of biofilm evolution become crucial.
Biofilm description

*Parts of biofilm structure:*
- Substratum
- Biofilm
- Boundary layer
- Bulk liquid

*Biofilm is a multiphase system:*
- Solid attached phase or biomass (cells)
- Suspended particles component (EPS)
- Liquid phase

*Biofilm formation:*
1) attachment of microorganisms (substrate) free floating in the bulk liquid
2) concentration gradient of this substrate towards the substratum
3) conversion of substrate into biomass
Problem formulation

- Array of sensors before the RO membrane
- Each one gets its own biofilm thickness
- Each one communicates just with immediate neighbors

**Objective:** estimate biofilm thickness in the RO membrane from the measurements at each node, in a total distributed way and based on local communications

\[ L_i \] Thickness of the biofilm at sensor “i”

\[ S_{B,i} \] Substrate concentration in the bulk liquid at sensor “i”

\[ S_{0,i} \] Substrate concentration at the substrate of sensor “i”

\( S_i(y) \) Substrate concentration as a function of \( y \) at sensor “i”

**Distributed implementation:**
- Scalability, robustness and flexibility
- Low range communication, less energy, suitable for time varying estimation
- Distributed control: each node takes its own decision to actuate (biocide dosing)
Mathematical modeling of biofilm evolution

\[
D \frac{\partial^2 S_i(y)}{\partial y^2} + r_i = 0
\]

Diffusion of substrate along the biofilm

\[
r_i = q_m \frac{S_i(y)}{S_i(y) + \Gamma} X_F
\]

Microbial mass given by Monod equat.

\[
\dot{j}_i = -D \frac{\partial S_i(y)}{\partial y}
\]

Flux in the biofilm given by Fick’s law

\[
\dot{j}_i = L_i q_m X_F
\]

Zero order and fully penetrated biofilm

\[
\frac{\partial S_i(y)}{\partial y} = -\frac{L_i q_m}{D} X_F
\]

Variation of the substrate concentration along the y coordinate at node i, as a function of the biofilm thickness \(L_i\), the biomass concentration \(X_F\), the diffusion coefficient \(D\) and the maximum specific substrate conversion rate \(q_m\)
Definition of the observation model

- Assuming $X_F$ constant along the $y$-coordinate, solution of the PDE is:

$$S_i(y) = S_{B,i} + \frac{q_m}{D} L_i^2 X_F - \frac{q_m}{D} L_i X_F y$$

- Particularizing for $y=0$:

$$L_i^2 = \left( \frac{\Delta_i D}{q_m} \right) X_F^{-1}$$, where $\Delta_i = S_{B,i} - S_{0,i}$

- Node $i$ gets the following noisy observation of the biofilm thickness:

$$\tilde{L}_i = L_i + w_i = \left( \frac{\Delta_i D}{q_m} \right)^{1/2} X_F^{-1/2} + w_i$$

- By doing $h_i = \left( \frac{\Delta_i D}{q_m} \right)^{1/2}$ the observation model becomes:

$$\tilde{L} = hX_F^{-1/2} + w$$
Distributed estimation of biomass concentration

- **Objective**: to compute an estimation \( \tilde{X}_F \) of the biomass concentration \( X_F \), from the noisy thickness estimations \( \tilde{L} \), such that every node can compute the biofilm thickness at the RO membrane \( \tilde{L}_m \):

\[
\tilde{L}_m = \left( \frac{\Delta_m D}{q_m} \right)^{1/2} \tilde{X}_F^{-1/2}
\]

- Given \( \tilde{L} \), the MLE of the parameter \( \theta = X_F^{-1/2} \) is computed by:

\[
\hat{\theta}_{ML} = \left( h^T h \right)^{-1} h^T \tilde{L}
\]

- This expression can be written as:

\[
\hat{\theta}_{ML} = (\alpha_1)^{-1} \alpha_2
\]

where each factor \( \alpha_1 = \sum_{i=0}^{N} h_i^2 \) and \( \alpha_2 = \sum_{i=1}^{N} h_i \tilde{L}_i \) can be computed in a distributed fashion by means of an iterative consensus process.
Residual gossip

- The nodes can compute the MLE by means of two iterative consensus processes:

  \[ \alpha_1 \]: each node “i” takes \( x_i(0) = h_i^2 \) as the initial value
  \[ \alpha_2 \]: each node “i” takes \( x_i(0) = h_i \tilde{L}_i \) as the initial value

- **Original asymmetric consensus:**
  \[
  \lim_{k \to \infty} x_i(k) = \alpha
  \]
  \[
  x_j(k + 1) = \epsilon x_i(k) + (1 - \epsilon)x_j(k), \quad \forall j \in \Omega_i(k)
  \]
  Keep the sum of the initial values along the iterations
  At each iteration, consider the residuals that are usually ignored

- **Asymmetric consensus plus residuals technique:**
  \[
  \lim_{k \to \infty} x_i(k) = \frac{1}{N} \sum_{i=1}^{N} x_i(0)
  \]
  \[
  x_j(k + 1) = \epsilon x_i(k) + (1 - \epsilon)(x_j(k) + r_i(k)), \quad \forall j \in \Omega_i(k)
  \]
  \[
  r_j(k + 1) = r_j(k) + x_j(k) - x_j(k + 1) + r_i(k), \quad \forall j \in \Omega_i(k)
  \]
Initial state

State of nodes: $4+3+7+9+1=24$
Residual of nodes: $0+0+0+0+0=0$
Summation of state plus residuals: 24
Node 1 sends a broadcast message

Sate of nodes: $4 + 3.5 + 5.5 + 6.5 + 2.5 = 22$
Residual of nodes: $0 - 0.5 + 1.5 + 2.5 - 1.5 = 2$
Summation of state plus residuals: 24
Node 3 sends a broadcast message

State of nodes: $5 + 4.75 + 5.5 + 6.25 + 2.5 = 24$
Residual of nodes: $-0.5 - 1.25 + 0 + 3.25 - 1.5 = 0$
Summation of state plus residuals: 24
Consensus reached

State of nodes: 4.8 + 4.8 + 4.8 + 4.8 + 4.8 = 24
Residual of nodes: 0 + 0 + 0 + 0 + 0 = 0
Summation of state plus residuals: 24
Numerical results

\[ \tilde{L}_m = \left( \frac{\Delta_m D}{q_m} \right)^{1/2} \tilde{X}_F^{-1/2} \]

Biofilm thickness at the RO membrane, as a function of the biomass concentration.

*MSE of the distributed estimator of the biomass concentration, when performed by either the residual gossip or the asymmetric gossip algorithms*
Conclusions

- Correct estimation of biofilm evolution is crucial for RO membrane maintenance.

- WSN’s suitable to estimate biofilm thickness at the RO membranes.

- Distributed estimator: scalability, flexibility, robustness, suitability for distributed control loop.

- Residual gossip:
  - The error inherent to asymmetric consensus is overcome.
  - More accurate estimation provided.
Numerical results

Real values of substrate concentration and biofilm thickness at each location along the sensor array

Noisy measurement of the biofilm thickness at each node, corrupted by observation error
Numerical results

Estimation of the biofilm thickness, computed at each node by using the residual gossip algorithm, and comparison with the real thickness.

\[ L_i^2 = \left( \frac{\Delta_i D}{q_m} \right) X_F^{-1} \]

Biofilm thickness at node \( i \), as a function of the biomass concentration.