Cell Coverage based on Social Welfare Maximization

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ABSTRACT

We introduce a new approach, based on social welfare maximization, for determining the coverage of a single cell in a WCDMA system. The approach takes into account resource usage in the uplink direction, and captures the tradeoff between having a large cell with many users versus having a smaller cell with fewer users, which however are able to transmit at a higher rate. Our numerical investigations demonstrate how cell coverage determined using the above approach is affected by various system parameters, such as the path propagation characteristics, the mobile power limit, and the distribution of mobile hosts within a cell.

I. INTRODUCTION

Due to their limited capacity, effective dimensioning of wireless networks in order to achieve efficient utilization of scarce resources is very important. Moreover, an important feature of Direct Sequence (DS) CDMA (Code Division Multiple Access), on which Wideband CDMA is based on, but also of other CDMA systems such as IS-95, is the ability to dynamically adjust the cell coverage area in order to achieve a balanced distribution of load amongst neighboring cells [1]. The above concept is known as dynamic cell management (DCM) or cell breathing.

The goal of traditional cell breathing is to evenly distribute the load amongst neighboring cells. Hence, cells under heavy load that experience high interference decrease their coverage, hence their load, whereas neighboring cells that happen to be less loaded increase their coverage, thus accommodate mobile users that couldn't be accommodated by the heavier loaded cell.

Load balancing is just one possible objective of dynamic cell management. We propose and investigate a procedure that has a different objective, namely that of maximizing the social welfare of a cell. The approach takes into account resource usage in the uplink direction, and captures the tradeoff between having a large cell with many users versus having a smaller cell with fewer users, which however are able to transmit at a higher rate. The social welfare is a microeconomic concept that represents the aggregate utility of a group of individuals, such as mobile users in a wireless network. The utility of each user represents how much he values the particular level of service he is receiving. The motivation and justification for seeking to maximize the social welfare comes from the fact that in a competitive environment, as is currently the case in the telecommunications market, the application of resource control and charging mechanisms based on social welfare maximization leads to more competitive services, and more efficient utilization of network resources based on actual user requirements.

A number of research works have considered the use of microeconomic models based on the notions of utility and pricing for the development of flexible and efficient control mechanisms in both wired [2,3], and wireless networks [4,5]. The work in [4,5] presents two approaches for power control in wireless networks based on the notion of user utility, and utilizing pricing as a feedback control mechanism. Our work differs from the above in that the proposed model takes into account the actual resource constraints in the uplink direction, and is applied to the problem of determining the cell coverage.

The rest of this paper is organized as follows. In Section II we discuss resource usage in the uplink direction of a CDMA system. In Section III we present a model for social welfare of a single CDMA cell, in the case of users with elastic (best-effort) traffic, which value only the average throughput of data transmission. In Section IV we present numerical investigations that demonstrate the application of our approach for determining the optimal cell coverage, and investigate how the approach is affected by various system parameters, such as the propagation characteristics, the mobile path transmission power limits, and the distribution of mobile users within a cell. Finally, Section V concludes the paper identifying ongoing and related research issues.

II. RESOURCE USAGE IN THE UPLINK

Consider the uplink of a single CDMA cell. The bitenergy-to-noise-density ratio at the base station is given by [6,7]

$$\left(\frac{E_b}{N_0}\right)_i = \frac{W}{r_i} \frac{g_i p_i}{\sum_{j \neq i} g_j p_j + \eta},$$
 (1)

where *W* is the chip rate, r_i is the transmission rate for mobile *i*, p_i is the transmission power for mobile *i*, g_i is the path gain between the base station and mobile *i*, and η is the power of the background noise at the

base station. The ratio W/r_i is the spreading factor or processing gain for mobile *i*.

The value of the bit-energy-to-noise-density ratio $(E_b / N_0)_i$ corresponds to the signal quality, since it determines the bit error rate, *BER* [6,7]. For additive white Gaussian noise, *BER* is a non-decreasing function of $(E_b / N_0)_i$. Let γ_i be the target bit-energy-to-noise-density ratio at the base station required to achieve a particular *BER*. This target is given to fast closed-loop power control, which adjusts the mobile transmission power in order to achieve it.

If we assume perfect power control, in which case $(E_b / N_0)_i = \gamma_i$, and solve the set of equations given by (1) for each mobile *i*, we get [7,8]

$$g_i p_i = \frac{\eta \alpha_i^{UL}}{1 - \sum_j \alpha_j^{UL}}, \qquad (2)$$

where the load factor α_i^{UL} is given by

$$\alpha_i^{UL} = \frac{1}{\left(\frac{W}{r_i \gamma_i} + 1\right)}.$$
(3)

Note that the power levels given by the set of equations (2) for $i \in I$, where *I* is the set of mobiles, are the minimum such that the target bit-energy-to-noise-density ratios $\{\gamma_i\}$ are met. Since the power p_i can take only positive values, from (2) we get

$$\sum_{i} \alpha_i^{UL} < 1.$$
 (4)

The last equation illustrates that the uplink is interference-limited: Even when they have no power constraints, mobile hosts cannot increase their power without bound, due to the increased interference they would cause to the other mobiles. If (4) is violated, then the target $\{\gamma_i\}$ cannot be met for all mobiles, and the system is infeasible.

When there is a large number of mobile users, each using a small portion of the available resources, we have $W/r_i\gamma_i >> 1$, hence $\alpha_i^{UL} \approx r_i\gamma_i/W$ and the resource constraint (4) can be approximated by

$$\sum_{i} r_i \gamma_i < W . \tag{5}$$

Up to now we have assumed that there are no constraints on the power a mobile can transmit with. When such power constraints exist, i.e., if mobile *i* can transmit with maximum power \overline{p}_i , then from (2) we get [8]

$$\sum_{i} \alpha_{i}^{UL} < 1 - \frac{\eta}{\min_{i} \left[\frac{g_{i} \overline{p}_{i}}{a_{i}^{UL}} \right]}.$$
 (6)

Hence, when there are power constraints, the total capacity is determined by one mobile host. Indeed, if all mobiles have the same power constraint and the same resource usage, then the total capacity is determined by the mobile with the smallest channel gain g_i , or equivalently by the mobile with the highest channel loss. Since loss is related to the distance from the base station, the uplink in this case is coverage-limited.

If we consider the approximation for a large number of mobiles, each using a small portion of the available resources, then (6) becomes

$$\sum_{i} r_{i} \gamma_{i} < W - \frac{\eta}{\min_{i} \left[\frac{g_{i} \overline{p}_{i}}{r_{i} \gamma_{i}} \right]}.$$
(7)

III. SOCIAL WELFARE

Next we discuss an expression for the utility of individual mobile users, and from that an expression for the social welfare of a single cell in a wireless system.

We consider the case of elastic (best-effort) traffic, where users value only the average throughput with which their data is successfully transmitted. The throughput is a product of the transmission rate and the probability of successful packet transmission. The latter is a function of the bit error rate *BER*, which as discussed above is a function of the target bit-energy-tonoise-density ratio γ . Hence, the probability of successful data transmission can be written as $P_s(\gamma)$, in which case the average throughput is $rP_s(\gamma)$ [4,9]. Thus, the utility for user *i* carrying elastic traffic has the form $U_i(r_i P_s(\gamma_i))$ (for simplicity we have assumed that the packet success rate is the same for all mobiles), hence the social welfare of a CDMA cell, given by the aggregate utility of all users, is

$$\sum_{i} U_i (r_i P_s(\gamma_i)). \tag{8}$$

One can prove that [10], if the utility $U_i(x_i)$ is increasing and strictly concave, then the maximum of (8), under the resource constraint (4) or (5) is achieved for a value of signal quality γ^* that depends only on $P_s(\gamma)$, i.e., it is independent of the user utilities. In particular, γ^* satisfies

$$P_{s}(\gamma^{*}) = P'_{s}(\gamma^{*})\gamma^{*}.$$
⁽⁹⁾

Moreover, the optimal rates $\{r_i^*\}$ can be found in a distributed and decentralized manner, by sending to each user a "price" signal that is proportional to the amount of wireless resources they use. From (1), this amount is given by the product $r_i\gamma_i$. Under the above approach, each user *i* will seek to maximize his net benefit, given by

$$U_i(r_i P_s(\gamma^*)) - \lambda r_i \gamma^*, \qquad (10)$$

where λ is the shadow price (price per unit of resource), and γ^* satisfies (9). The transmission rate r_i^* that maximizes the user's net utility given by (10) satisfies the following

$$U'_{i}\left(r_{i}^{*}P_{s}(\gamma^{*})\right)P_{s}(\gamma^{*})=\lambda\gamma^{*}.$$

One approach for finding the shadow price is to start from a small value, and gradually increase it as long as the resource constraint (4) or (5) is violated.

IV. CELL COVERAGE BASED ON SOCIAL WELFARE MAXIMIZATION

As discussed in the previous section, when there are constraints on the maximum power with which each mobile can transmit, the coverage will be limited by the mobile experiencing the largest path loss. However, by decreasing the coverage, i.e., by not serving mobiles that are far from the base station and experience heavy path loss, the remaining mobiles would be able to send with higher power and achieve a higher transmission rate, hence the aggregate utility of the remaining mobile users could increase.

In this section we investigate the aforementioned tradeoff between coverage and social welfare, and how it is affected by various parameters of the system. Note that we consider a single cell, and do not take into account the interference caused by neighboring cells.

A. System Parameters

The propagation model considered is the Okumara-Hata model [11], which for an urban environment gives

$$L_{urb}(d) = 137.4 + 35.2 \log_{10}(d)$$
,

where L(d) is the path loss in dB and d is the mobile's distance from the base station in Km. For a suburban environment, the model is

$$L_{sub}(d) = 129.4 + 35.2 \log_{10}(d)$$

Regarding the distribution of users within a cell, we assume that the number of mobiles within a radius of d (in Km) from the base station is

$$N(d) = \rho_N \pi d^{\nu}.$$

In the numerical investigations we consider the values $\rho_N = 10,20$, and 30, and $\upsilon = 1,1.5$, and 2; the value $\upsilon = 2$ corresponds to the case where the density of mobile users within the cell is constant, independent of the distance from the base station.

Next we describe the model for the packet success rate $P_s(\gamma)$. In the case of additive white Gaussian noise and a non-fading channel, the bit error rate for DPSK (Differential Phase Shift Keying) modulation is [12]

$$BER(\gamma) = 0.5e^{-\gamma}$$

If there is no error correction, and bit errors are independent and are all detected, then the packet success rate is given by

Parameter	Value
Noise, η	10^{-13} Watt
Max. mobile power, \overline{p}	0.2, 0.6 Watt
Path gain, $g(d)$	$kd^{-u}, u = 3.52,$
	$k_{urb} = 1.82 \cdot 10^{-14}$,
	$k_{sub} = 1.15 \cdot 10^{-13}$
# of mobiles, $N(d)$	$\rho_N \pi d^{-\upsilon}$,
	$\rho_N = 10,20,30, \upsilon = 1,1.5,2$
$BER(\gamma)$ (DPSK)	$0.5e^{-\gamma}$
# of bits per pkt, M	60
γ^*	5
Utility, $U(x)$	$1 - e^{-bx}$, $b = 0.1, 0.2$

Table 1: Parameters for the numerical investigations.

$$P_{s}(\gamma) = \left(1 - BER(\gamma)\right)^{M},$$

where *M* is the number of bits in one packet. From the above equation, and with (9) the optimal bit-energy-to-noise-density ratio is found to be $\gamma^* = 5$.

Note that expressions for the packet success rate can be derived for other modulation schemes and in the presence of forward error correction (FEC) [10].

The values of the other parameters that are used in the numerical investigations are shown in Table 1. For simplicity, we assume that all mobile users have the same utility and the same maximum power constraint.

B. Results and Discussion

Figure 1 shows the social welfare as a function of the coverage d. Observe that the dependence is concave, and the social welfare is maximized at some coverage. Furthermore, the figure shows the effect of the maximum transmission power. In particular, for a higher maximum transmission power, the social welfare increases, and the social welfare maximum is achieved for a larger coverage. The reason for such behavior is that an increasing power limit increases the right-hand side of the resource constraints (6) and (7). Indeed, a larger power limit enables mobiles to achieve a higher transmission rate, as shown in Figure 3.

Figure 2 shows the social welfare as a function of coverage, and how it is affected by the path propagation characteristics. For a suburban environment where the path gain is higher than in an urban environment, both the social welfare and the coverage at which the social welfare is maximized are higher, compared to an urban environment. As with the mobile power limit, the reason for such behavior is that increasing the path gain increases the right-hand side of (6) and (7). Moreover, in a suburban environment mobiles can achieve a higher transmission rate, Figure 3.

Figure 4 shows that for a steeper utility, i.e., a utility with larger *b* (see Table 1), both the social welfare and the coverage at which the social welfare is maximized is higher. The reason for this behavior is that if utility U_1 is steeper than U_2 , we have $U_1(x) > U_2(x)$, i.e., for a steeper utility, the same amount of wireless resources $x (= r\gamma)$ yields a high user satisfaction, hence increasing the coverage to accommodate more mobile users, even if that means giving each less resources, results in a higher social welfare, compared to that achieved with the less steep utility.

Figure 5 shows that when the density of mobile users decreases with the distance from the base station (this corresponds to the case v = 1 in Table 1), the social welfare remains approximately the same with the case where the density of mobile users is independent of the distance from the base station (this corresponds to the case v = 2 in Table 1); hence the distribution of mobile users, for the parameter values considered, has a small effect on the value of the social welfare. However, the coverage for which the social welfare is maximized is larger in the latter case (v = 2). In other words, if the increase of coverage increases the number of mobile users that can be accommodated by a significant amount, then this can result in higher social welfare.



Figure 1: Social welfare as a function of coverage and effect of mobile power limit.



Figure 2: Social welfare as a function of coverage and effect of path gain.



Figure 3: Rate as a function of coverage and effect of mobile power limit and propagation characteristics.



Figure 4: Social welfare as a function of coverage and effect of user utility.



Figure 5: Social welfare as a function of coverage and effect of mobile user distribution.



Figure 6: Social welfare as a function of coverage and effect of distribution density factor.

Finally, Figure 6 shows that the distribution density factor (ρ_N in Table 1) does not affect the coverage at which the social welfare is maximized, but it does affect the value of the social welfare maximum.

V. CONCLUSIONS

We have presented a new approach for determining the coverage of a single cell in a CDMA wireless system, which takes into account the requirements of mobile users. This will be important in future wireless systems, which will support data applications with various performance requirements. The approach is based on maximizing the social welfare of the system, and considers the tradeoff between having a large cell, hence accommodating a large number of mobile users, versus having a smaller cell, hence accommodating a smaller number of users, which however can achieve a higher transmission rate.

Ongoing work is investigating the extension of the approach to a multiple cell environment; in such an environment, the transmission in one cell causes interference to other neighboring cells. A further issue for investigation is the comparison of the approach with other schemes for dynamic cell management and cellular network planning, as well as the integration of the approach with other network planning procedures, which take into account issues such as fast fading, shadowing, and power control errors.

The work presented in this paper is part of a wider effort whose goal is to investigate the application of ideas from microeconomic modeling for developing flexible, efficient, and robust procedures for resource control and management in wireless networks. In this direction, issues we are currently investigating include the following:

- Procedures for selecting the target bit-energy-tonoise-density ratio; these are part of open-loop power control.
- Congestion-sensitive downlink power control.

- Integration of control mechanisms for wireless and wired networks.
- Resource control and service differentiation for WLANs based on 802.11.

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