

# Congestion Pricing for Resource Control in Wideband CDMA

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Congestion pricing has been identified as a flexible mechanism for efficient and robust resource control in fixed wireline networks. In this report we investigate the application of congestion pricing in Wideband CDMA (Code Division Multiple Access) networks. We begin by discussing the problem of quantifying resource usage for both the uplink and the downlink in CDMA networks. In the uplink, resource usage is an increasing function of the product of the transmission rate and the signal quality, given by the target bit-energy-to-noise-density ratio. In the downlink, resource usage is given by the transmission power from the base station. Based on these results, we propose and analyze a congestion pricing framework for elastic traffic in WCDMA networks, identifying how the properties of wireless networks affect the application of congestion pricing models.

Our framework seeks to exploit the joint control of the transmission rate and signal quality in order to achieve, in a distributed and decentralized manner, economically efficient utilization of wireless network resources. The framework incorporates the congestion charge for shared resources in the wireless and wireline network, as well as the cost of battery power consumption at the mobile hosts, and is extended to the case of hybrid code and time division scheduling. For elastic traffic we show that the net utility maximization problem can be decomposed into two simpler problems: one involving the selection of the optimal target bit-energy-to-noise-density ratio, and one involving the selection of the optimal transmission rate. This result can simplify the integration of rate control at the CDMA and the transport layer. Whereas the above considered resource control on timescales larger than those of fast closed-loop power control, our final investigation involves the application of congestion pricing for achieving fair, efficient, and robust power control.

**Keywords:** wireless data networks, resource usage, rate control, power control, elastic traffic, utility, social welfare



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# 1 Introduction

Procedures for efficient control and management of wireless network resources are becoming increasingly important. This is due to two factors: First, there is a limited ability, compared to fixed wireline networks, for increasing the capacity of mobile wireless networks. Second, emerging multimedia services and applications will increase the demand for bandwidth in wireless networks. Congestion pricing has been identified as a flexible mechanism for efficient and robust resource control in fixed wireline networks [17, 21, 4, 18, 5]. In the work presented in this report we investigate the application of congestion pricing in wireless WCDMA (Wideband Code Division Multiple Access, CDMA) networks.

WCDMA has emerged as the most widely adopted third generation (3G) air interface technology [12]. With FDMA (Frequency Division Multiple Access) each mobile uses a difference portion of the radio spectrum, and with TDMA (Time Division Multiple Access) each mobile can use the shared radio resource only in the time slots it has been allocated. On the other hand, with CDMA all mobile hosts can simultaneously use the whole radio spectrum, and unique digital codes are used to differentiate the signal from different mobiles; such an approach enables simpler statistical multiplexing, without the need for complex time or frequency scheduling mechanisms. WCDMA is based on Direct Sequence CDMA (DS-SS), a spread spectrum technology where the user data bits are spread over the entire spectrum used for transmission, Figure 1, which for WCDMA is approximately 5 MHz in each direction<sup>1</sup> (uplink and downlink). An important advantage of WCDMA is the support for variable bit rates, that is achieved with the use of variable spreading factors and multiple codes. Finally, all the cells in a WCDMA network use the same frequency spectrum; this feature is behind the soft-capacity property of WCDMA networks, which results in the graceful degradation of performance as the load increases. Note that although our discussion is focused on WCDMA, including its code and time division scheduling modes, our results are more generally applicable to CDMA-based wireless systems.

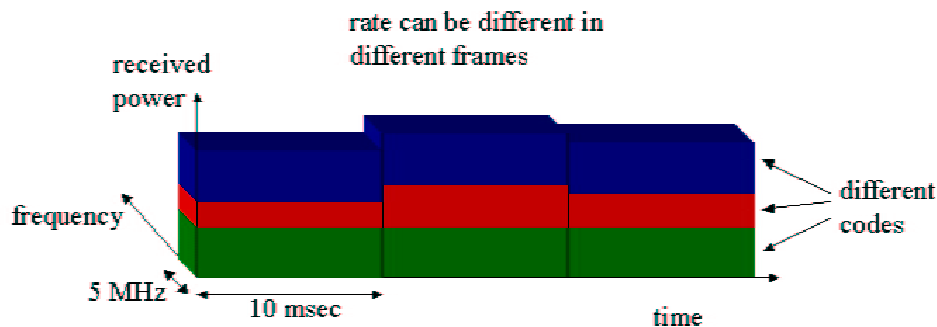


Figure 1: In WCDMA, data is spread over the entire spectrum. Signals to and from different mobiles are identified with unique codes. Transmission in WCDMA occurs in fixed-size frames, the minimum duration of which is 10 milliseconds. The rate can change between frames, but must remain the same within a single frame.

In this report, we first investigate the problem of quantifying usage of shared resources in CDMA networks. The shared resources in such networks include the radio spectrum and the power at the base station. From this investigation, we find that resource usage in the uplink is an increasing function of the product of

<sup>1</sup>This is the case for WCDMA's Frequency Division Duplex (FDD) mode. In the Time Division Duplex (TDD) mode it uses a single 5 MHz band for both directions.

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the transmission rate and the signal quality, the latter expressed in terms of the target bit-energy-to-noise-density ratio ( $E_b/N_0$ ). In the downlink, resource usage is determined by the transmission power; for a given transmission rate, the power together with the path loss characteristics and the orthogonality of the downlink codes, determine the received signal quality, i.e., the ratio  $E_b/N_0$ . Building on the results for resource usage, the main findings and contributions of our work are summarized as follows:

- We propose expressions for a mobile user's utility that are appropriate for elastic traffic, which can adapt both its transmission rate and target signal quality, and for rate-inelastic traffic, which has fixed requirements in terms of transmission rate, but can adapt its signal quality.
- We describe and analyze a framework for resource control, for both the uplink and the downlink direction in wireless networks; the framework incorporates the congestion charge for wireless and wireline network resources, as well as the cost of battery power consumption at mobile hosts. Within this framework we discuss two approaches for the application of congestion pricing: In the first, prices are explicitly communicated to mobile users, which then react by adjusting their transmission rate. In the second, mobile users communicate a willingness-to-pay to the radio network controller (RNC), which allocates shared resources in proportion to the willingness-to-pay values of all users.
- In the downlink, resource usage is determined by the transmission power. In addition to a model where congestion charges are proportional to the transmission power, we discuss an alternative model where charges are, similar to the uplink, proportional to the transmission rate and the signal quality. An important feature of the latter model is that charges are directly related to quantities that reflect the performance a mobile user is experiencing, and it does not differentiate users based on their location.
- We show that, in the case of elastic traffic where users value only their average throughput, the selection of the optimal signal quality that maximizes a user's benefit, i.e., his utility minus charge, is independent of the user's utility and of the congestion price, and depends only on the packet error probability as a function of signal quality. This result is important since it greatly simplifies the user's price reaction process by decoupling the two problems: the selection of the target signal quality and the selection of the transmission rate; this decoupling enables the integration of the congestion loop at the CDMA layer<sup>2</sup> with that at the transport layer. Moreover, the selection of optimal  $E_b/N_0$  needs to be performed at the start of data transmission, or whenever the dependence of the packet error probability on  $E_b/N_0$  changes.
- We extend the model from pure code division scheduling, to hybrid code and time division scheduling, both supported by WCDMA, and investigate the aggregate utility (social welfare) when using the combination of the two scheduling schemes. This extension includes quantifying resource usage in hybrid code/time division scheduling and defining a utility for mobile users that takes into account how much they value being able to continuously send data, which is possible with code division multiplexing, as opposed to sending data in bursts, which is the case with time division multiplexing.
- Finally, based on the congestion pricing model for rate-inelastic traffic, we propose and investigate an approach for fair, efficient, and robust power control

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<sup>2</sup>We will use the term "CDMA layer" to refer to all the layer of a CDMA network below the data link layer.

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for the downlink in WCDMA networks. Important properties of the approach are that it takes into account the actual resource usage of each user, and gives incentives for users to back-off in cases of high congestion.

Our work differs from other works, which are reviewed in Section 6, in that it incorporates both the congestion charges for resources in wireless and wireline networks, as well as the cost due to battery power consumption. Such a modelling approach can enable the integration of resource control at the CDMA layer and the transport layer. On the other hand, the vast majority of work that consider the application of microeconomic approaches to resource control in wireless networks have focused exclusively on the cost of battery power. This was motivated by the fact that battery power was, and continues to be, a scarce resource in mobile hosts. In future 3G wireless networks supporting multimedia services with high bandwidth requirements, wireless resources are also likely to be scarce, hence in addition to the cost of battery power, it will be important to incorporate the congestion of wireless resources in power and rate control procedures.

The rest of this report is organized as follows. In Section 2 we investigate resource usage for both the uplink and downlink direction of CDMA networks. In Section 3 we propose and investigate a framework for congestion pricing of elastic traffic in CDMA networks. In particular, in Section 3.1 we discuss the utility for elastic traffic in wireless networks, in Section 3.2 we discuss the uplink, and in Section 3.3 we discuss the downlink. In Section 4 we extend the congestion pricing framework to hybrid code division/time division scheduling. In Section 5 we investigate a model for congestion pricing of rate-inelastic traffic that has fixed requirements in terms of the transmission rate, but is elastic in terms of its target signal quality; based on this model we propose a fair, efficient, and robust power control algorithm for the downlink in CDMA networks. In Section 6 we review related work, identifying where it differs from our work. Finally, in Section 7 we outline some outstanding and related research issues.

## **2 Resource usage in CDMA networks**

In this section we investigate resource usage for the uplink and downlink in CDMA networks, identifying the key parameters that affect resource usage in each direction.

Consider a single CDMA cell. Let  $W$  be the chip rate, which is fixed and equal to 3.84 Mcps for WCDMA. The bit-energy-to-noise-density ratio,<sup>3</sup>  $E_b/N_0$ , at a receiver (either the mobile host or the base station) is given, in the case of matched filter receivers,<sup>4</sup> by [6, 47, 10]

$$\left(\frac{E_b}{N_0}\right)_i = \frac{W}{r_i} \frac{g_i p_i}{I_i + \eta_i}, \quad (1)$$

where  $r_i$  is the transmission rate,  $p_i$  is the transmission power,  $g_i$  is the path gain between the base station and mobile  $i$ ,  $I_i$  is the power of the interference, and  $\eta_i$  is the power of the background noise. The path gain depends on channel imperfections such as attenuation, shadowing, and multipath fading.

The value of the bit-energy-to-noise-density ratio  $(E_b/N_0)_i$  corresponds to the signal quality, since it determines the bit error rate,  $BER$  [6, 47]. Due to the errors in the wireless network, the actual throughput, i.e., rate of successful data delivery, will be smaller than  $r_i$ . Under the realistic assumption of additive white Gaussian

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<sup>3</sup>A note regarding terminology: The term signal-to-interference-plus-noise (or simply signal-to-interference) ratio SINR (or SIR) is sometimes used for what we defined as  $E_b/N_0$ ; however, this is not universal, since SINR can also be taken to be  $\frac{r}{W} \frac{E_b}{N_0}$ , i.e., the carrier signal to interference ratio.

<sup>4</sup>Use of multiuser detection can help decrease the interference from other user signals.

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noise,  $BER$  is a non-decreasing function of  $E_b/N_0$  that depends on the multipath characteristics, and the modulation and forward error correction (FEC) algorithms. Let  $\gamma_i$  be the target bit-energy-to-noise-density ratio required to achieve a target<sup>5</sup>  $BER$ . This target is given to fast closed-loop power control<sup>6</sup>, which adjusts the transmission power in order to achieve it. If we assume perfect power control, then  $(E_b/N_0)_i = \gamma_i$ .

The ratio  $W/r_i$  is the spreading factor or processing gain for mobile  $i$ . From (1), observe that for a higher spreading factor, equivalently a smaller transmission rate, the same target  $E_b/N_0$  will be achieved with less power. Variable bit rate transmission can be supported with codes corresponding to different spreading factors, while keeping the chip rate the same, and with the use of multiple codes. In WCDMA [12] transmission occurs in frames with a minimum duration of 10 milliseconds; the rate is allowed to change between frames, but remains the same within a single frame.

The spreading factor on the uplink dedicated channel are powers of 2 and can range from 256, giving a channel bit rate<sup>7</sup> of 15 Kbps, to 4, giving a channel bit rate of 960 Kbps; higher bit rates are achieved by using up to 6 parallel codes with spreading factor 4 (giving a channel bit rate of 5740 Kbps). In the downlink, the spreading factor can range from 512 to 4. Moreover, in the downlink, orthogonal codes are selected according to the maximum transmission rate.

When a sender does not send data continuously, the average  $E_b/N_0$  requirements will be met, if the right hand-side of (1) is multiplied by the percentage of time the sender is "on", i.e., actually transmitting data; this percentage is called activity factor, and for voice is 0.67.

### 2.1 Uplink

In the uplink, the interference  $I_i$  for mobile  $i$  is the sum of the power of the signals received by the base station from all other mobile hosts within the same cell, i.e.,  $I_i = \sum_{j \neq i} g_j p_j$ . Moreover, we can assume that the background noise at the base station is the same for all mobiles, i.e.,  $\eta_i = \eta$ . If  $\gamma_i$  is the target bit-energy-to-noise-density ratio, then under perfect power control  $(E_b/N_0)_i = \gamma_i$ , (1) becomes

$$\gamma_i = \frac{W}{r_i} \frac{g_i p_i}{\sum_{j \neq i} g_j p_j + \eta}. \quad (2)$$

Solving the set of equations given by (2) for each mobile  $i$ , we get [47, 38, 9]

$$g_i p_i = \frac{\eta \alpha_i^{\text{UL}}}{1 - \sum_j \alpha_j^{\text{UL}}}, \quad (3)$$

where the load factor  $\alpha_i^{\text{UL}}$  is given by

$$\alpha_i^{\text{UL}} = \frac{1}{\left(\frac{W}{r_i \gamma_i} + 1\right)}. \quad (4)$$

Note that the power levels given by the set of equations (3) for  $i \in I$ , where  $I$  is the set of mobiles, are the minimum such that the target bit-energy-to-noise-density

<sup>5</sup>The target signal quality can also be expressed in terms of the block error rate,  $BLEER$ , or the frame error rate,  $FER$ . In practise up to know, the target signal quality is set the same for all users. Nevertheless, it has been identified that differentiated service can be offered by setting a different target for different users.

<sup>6</sup>WCDMA supports fast (1500 Hz) closed-loop power control in both the uplink and the downlink. On the other hand, IS-95, a second generation narrowband CDMA system, supports fast (800 Hz) closed-loop power control only in the uplink.

<sup>7</sup>The maximum user data rate with 1/2 rate coding is approximately half the channel bit rate.

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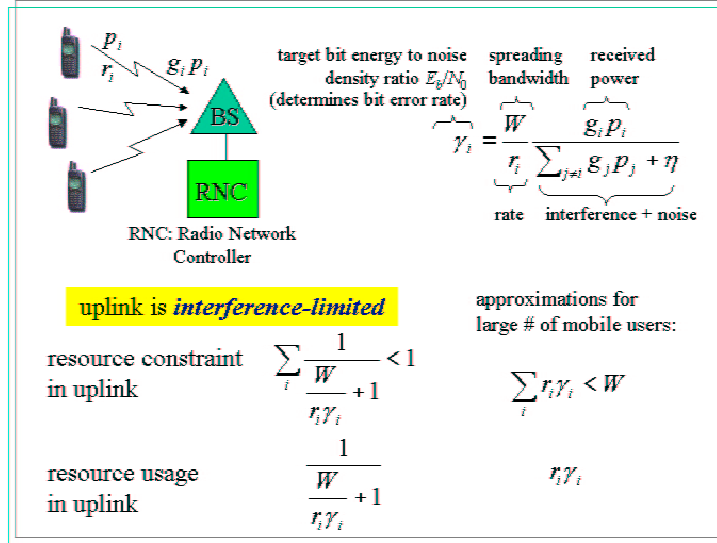


Figure 2: The uplink is interference-limited, when there are no constraints on the maximum transmission power from the mobile hosts. Resource usage is an increasing function of the product of the rate  $r_i$  and the target bit-energy-to-noise-density ratio  $\gamma_i$ . For a large number of mobiles, resource usage can be approximated by the product  $r_i \gamma_i$ . When there are power limits in the mobile hosts, the uplink is coverage-limited.

ratios  $\{\gamma_i\}$  are met. Since the power  $p_i$  can take only positive values, from (3) we get

$$\sum_i \alpha_i^{\text{UL}} < 1. \quad (5)$$

The sum in (5) is called uplink load factor. Moreover, (5) illustrates that the uplink is *interference-limited*: Even when they have no power constraints, mobile hosts can not increase their power with no bound, due to the increased interference they cause to the other mobiles. If (5) is violated, then the target  $\{\gamma_i\}$  can not be met for all mobiles, and the system is infeasible.

Moreover, (5) suggests that  $\alpha_i^{\text{UL}}$  is a measure of the resource usage or the “effective usage” of a mobile host  $i$ , in the uplink direction. Observe from (4) that resource usage in CDMA networks is determined by two parameters, which can be controlled independently: the transmission rate  $r_i$  and the signal quality, expressed in terms of the target bit-energy-to-noise-density ratio  $\gamma_i$ ; moreover, resource usage is an increasing function of their product  $r_i \gamma_i$ . The above result was for the case of linear single user (matched filter) receivers; expressions for resource usage can also be defined for multiuser receivers [11].

A useful expression for measuring the uplink load factor  $\sum_i \alpha_i^{\text{UL}}$  can be found by summing (3) for all mobiles:

$$\begin{aligned} \sum_i g_i p_i &= \frac{\eta \sum_i \alpha_i^{\text{UL}}}{1 - \sum_j \alpha_j^{\text{UL}}} \Rightarrow \\ \sum_i \alpha_i^{\text{UL}} &= \frac{I_{\text{total}} - \eta}{I_{\text{total}}}, \end{aligned} \quad (6)$$

where  $I_{\text{total}}$  is the total received power, including the noise power. Hence, the estimation of the uplink load factor requires measurements of the total interference and the noise, both of which can be performed at the base station.

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### 2.1.1 Large number of mobile hosts

When there are a large number of mobile users, each using a small portion of the available resources, we have  $\frac{W}{r_i \gamma_i} \gg 1$ , hence  $\alpha_i^{\text{UL}} \approx \frac{r_i \gamma_i}{W}$  and the resource constraint (5) can be approximated by

$$\sum_i r_i \gamma_i < W. \quad (7)$$

If multiple codes are used, which is the case in WCDMA when transmission rates higher than 1 Mbps are required, then the total resource usage is  $\sum_{k=1}^K r_{ki} \gamma_{ki}$ , where  $K$  is the number of codes used by mobile  $i$  and  $r_{ki}, \gamma_{ki}$  is the rate and the target bit-energy-to-noise-density ratio for code  $k$ , respectively.

### 2.1.2 Power constraints and coverage

Up to now we have assumed that there are no constraints on the power a mobile can transmit. In the case there are such power constraints, namely if mobile  $i$  can transmit with maximum power  $\bar{p}_i$ , then from (3) we get [38]

$$\sum_i \alpha_i^{\text{UL}} \leq 1 - \frac{\eta}{\min_i \left[ \frac{g_i \bar{p}_i}{\alpha_i^{\text{UL}}} \right]}. \quad (8)$$

Hence, when there are power constraints, the total capacity is determined by one mobile host. Indeed, if all mobiles have the same power constraint and the same resource usage, then the total capacity is determined by the mobile with the smallest channel gain  $g_i$ , equivalently the highest channel loss. Since loss is related to the distance from the base station, the uplink in this case is *coverage-limited*.

Hence, from the above we see that the coverage of a CDMA cell is determined by the constraint on the uplink load factor: a smaller constraint results in a larger coverage. In radio network planning this constraint is expressed in terms of the interference margin or noise rise,  $I_{\text{margin}}$ , which is given by the ratio of the total received power (including the noise) divided by the noise power:

$$I_{\text{margin}} = \frac{I_{\text{total}}}{\eta},$$

in which case the constraint on the total load becomes

$$\sum_i \alpha_i^{\text{UL}} \leq \frac{I_{\text{margin}} - 1}{I_{\text{margin}}}.$$

The above model can be extended to the case where we have two (or more) traffic classes, e.g., real-time and non real-time, for which there is a bound on the percentage of the capacity used by one class (e.g., non real-time) or the total power of the signals received at the base station from one traffic class, thus limiting the interference that this class causes to the other (e.g., real-time).

### 2.1.3 Multiple cells

The above results were for the case of a single cell. In the case of multiple cells, the results can be generalized by including the intercell interference coefficient  $f = (\text{other cell interference})/(\text{intracell interference})$  [6, 12]. In this case, (2) becomes

$$\gamma_i = \frac{W}{r_i} \frac{g_i p_i}{(1+f) \sum_j g_j p_j - g_i p_i + \eta},$$

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from which we get

$$(1 + f) \sum_i \alpha_i^{\text{UL}} < 1.$$

Typical values for the interference coefficient are  $f \in [0.1, 0.6]$ .

The last equation illustrates the “soft capacity” property of CDMA networks, namely that the capacity of a cell depends on the load of neighboring cells: when its neighboring cells have a low load, the cell has a higher capacity, whereas if its neighboring cells have a high load, then it has less capacity.

Note that in the special case of network-wide soft handover (macrodiversity), the capacity of the whole wireless network can be completely characterized by [10]

$$\sum_i \alpha_i^{\text{UL}} < K,$$

where  $K$  is the number of base stations in the network.

### 2.2 Downlink

In the downlink, the total interference for mobile  $i$  is given by  $I_i = \theta_i g_i \sum_{j \neq i} p_j$ , where  $\theta_i$  represents the orthogonality of the codes used in the downlink<sup>8</sup>,  $g_i$  is the channel gain from the base station to mobile  $i$ , and  $p_j$  is the transmission power to mobile  $j$ . If  $\gamma_i$  is the target signal quality for mobile  $i$ , and assuming, as in the previous sections, that we have perfect power control, then (1) becomes

$$\gamma_i = \frac{W}{r_i} \frac{g_i p_i}{\theta_i g_i \sum_{j \neq i} p_j + \eta_i}. \quad (9)$$

The orthogonality factor  $\theta_i$  depends on multipath effects, hence can be different for different mobile hosts. Typical values fall in the range  $[0.1, 0.6]$ , see [12, p. 163].

In the downlink, unlike the uplink, there is a limit on the total transmission power<sup>9</sup>, say  $\bar{p}$ , hence the downlink is *power-limited*. The corresponding resource constraint is given by

$$\sum_i p_i \leq \bar{p}. \quad (10)$$

The last equation suggests that the transmission power from the base station characterizes resource usage in the downlink direction.

Next we derive the expression for the total power constraint in terms of  $r_i, \gamma_i$ . From (9), we get

$$\begin{aligned} \frac{W}{r_i \gamma_i \theta_i} p_i &= \sum_j p_j - p_i + \frac{\eta_i}{\theta_i g_i} \Rightarrow \\ \left(1 + \frac{W}{r_i \gamma_i \theta_i}\right) p_i &= \sum_j p_j + \eta'_i \Rightarrow \\ p_i &= \alpha_i \sum_j p_j + \alpha_i \eta'_i, \end{aligned} \quad (11)$$

where we have substituted  $\eta'_i = \frac{\eta_i}{\theta_i g_i}$  and  $\alpha_i = \frac{1}{\frac{W}{r_i \gamma_i \theta_i} + 1}$ .

<sup>8</sup>WCDMA employs orthogonal codes in the downlink. Due to multipath propagation, however, the mobile will receive part of the base station signal as multiaccess interference. On the other hand, multipath propagation can increase the power of the received signal. Which of the two effect is larger depends on the distance of the mobile from the base station, and its speed; see [12, p. 255-257]. In the uplink, transmission is asynchronous, hence the signals are not orthogonal.

<sup>9</sup>The total transmission power here refers to the total power the base station can transmit minus the power used for the downlink control channels.

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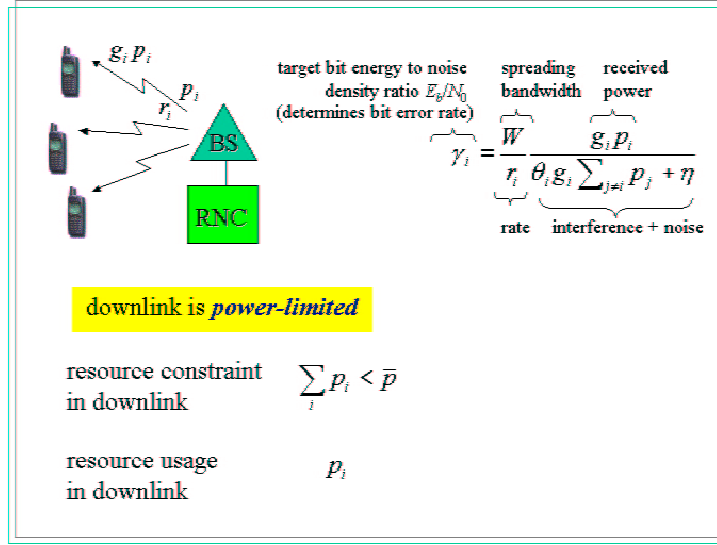


Figure 3: The downlink is power-limited. Resource usage characterized by the transmission power  $p_i$ .

Summing (11) for all mobiles gives

$$\begin{aligned}
 \sum_i p_i &= \sum_i \alpha_i \sum_j p_j + \sum_i (\alpha_i \eta'_i) \Rightarrow \\
 (1 - \sum_i \alpha_i) \sum_i p_i &= \sum_i (\alpha_i \eta'_i) \Rightarrow \\
 \sum_i p_i &= \frac{\sum_i (\alpha_i \eta'_i)}{1 - \sum_i \alpha_i}. \tag{12}
 \end{aligned}$$

Observe in the last equation that as  $\sum_i \alpha_i$  approaches 1, the total power required at the base station tends to infinity.

From (10) and (12) we get

$$\begin{aligned}
 \frac{\sum_i (\alpha_i \eta'_i)}{1 - \sum_i \alpha_i} &\leq \bar{p} \Rightarrow \\
 \sum_i (\alpha_i \eta'_i) &\leq \bar{p} - \bar{p} \sum_i \alpha_i \Rightarrow \\
 \sum_i (\alpha_i \eta'_i) + \bar{p} \sum_i \alpha_i &\leq \bar{p} \Rightarrow \\
 \sum_i [\alpha_i (\eta'_i + \bar{p})] &\leq \bar{p} \Rightarrow \\
 \sum_i \left[ \alpha_i \left( \frac{\eta'_i}{\bar{p}} + 1 \right) \right] &\leq 1. \tag{13}
 \end{aligned}$$

Equation (13) suggests that resource usage in the downlink direction can be expressed in terms of the rate  $r_i$  and target bit-energy-to-noise-density ratio  $\gamma_i$  by

$$\alpha_i^{\text{DL}} = \alpha_i \left( \frac{\eta'_i}{\bar{p}} + 1 \right) = \frac{\frac{\eta_i}{\theta_i g_i \bar{p}} + 1}{\frac{W}{r_i \gamma_i \theta_i} + 1}. \tag{14}$$

Unlike the uplink, where resource usage is given by (4), resource usage in the downlink is not independent of the path gain, hence of the mobile's position.

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### 2.2.1 Large number of mobile hosts

When there is a large number of mobiles,  $\frac{W}{r_i \gamma_i \theta_i} \gg 1$ , hence (14) becomes

$$\alpha_i^{\text{DL}} \approx \frac{(\frac{\eta_i}{\theta_i g_i \bar{p}} + 1) r_i \gamma_i \theta_i}{W}, \quad (15)$$

in which case the constraint (13) can be approximated by

$$\sum_i \left( \frac{\eta_i}{g_i \bar{p}} + \theta_i \right) \frac{r_i \gamma_i}{W} \leq 1.$$

If we consider average values for  $\eta_i$ ,  $l_i = 1/g_i$ , and  $\theta_i$ , as is frequently considered in downlink dimensioning, e.g., see [12, p. 163-165], then from the last equation we have

$$\sum_i r_i \gamma_i \leq \frac{W \bar{p}}{\bar{l} \bar{\eta} + \theta \bar{p}}. \quad (16)$$

### 2.2.2 Multiple cells

In the case of multiple cells, we can consider the intercell interference coefficient  $f_i$ , which now can differ for different mobiles since it depends on its position within the cell. If only the average value is considered, then in (16) we can replace  $\theta$  with  $(\bar{\theta} + \bar{f})$ .

## 3 Congestion pricing for elastic traffic

In this section we first propose a utility function that is appropriate for elastic traffic in wireless networks, which can adapt both its transmission rate and its signal quality. Then, based on the results for resource usage of the previous section, we present and investigate congestion pricing models for the uplink and downlink in CDMA networks.

### 3.1 Utility for elastic traffic

In the case of elastic (best-effort) traffic, users value the average throughput with which their data is successfully transmitted. The throughput is a product of the transmission rate and the probability of successful packet transmission. The latter is a function of the bit error rate *BER*, which as discussed in the beginning of Section 2, is a function of the target bit-energy-to-noise-density ratio  $\gamma$ . Hence, the probability of successful packet<sup>10</sup> transmission can be written as  $P_s(\gamma)$ , in which case the average throughput is  $rP_s(\gamma)$  [33, 44, 7]. Thus, the utility for elastic traffic where users value only their average throughput has the form

$$U(rP_s(\gamma)).$$

If the mobile user does not have minimum rate requirements, then the utility is typically concave. On the other hand, in the case the user has minimum rate requirements, equivalently maximum delay requirements, the utility can have a sigmoid shape. In the numerical examples of Appendix B.2, we consider both forms of the utility.

Note that the packet success probability  $P_s(\gamma)$ , in addition to the modulation and error correction algorithms, depends on a path's multipath characteristics, hence can be different for different mobiles; see [12, p. 196-198, 231], [49, p. 43-48].

<sup>10</sup>Packet here refers to the data unit over which error detection is performed.

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Let  $c(r_i, \gamma_i, p_i)$  be the charge incurred by user  $i$  with rate  $r_i$ , target bit-energy-to-noise-density ratio  $\gamma_i$ , and transmission power  $p_i$ . The net utility maximization problem for the user has the following general form (unless otherwise noted, we assume that the packet success probability  $P_s(\gamma)$  is the same for all users):

$$\begin{aligned} &\text{maximize} && U_i(r_i P_s(\gamma_i)) - c(r_i, \gamma_i, p_i) && (17) \\ &\text{over} && r_i \geq 0, \gamma_i \geq 0, \end{aligned}$$

where the variables  $r_i, \gamma_i, p_i$  are related by (2) or (9), which have the generic form

$$\gamma_i = \frac{W}{r_i} \frac{\phi_{ii} p_i}{\sum_{j \neq i} \phi_{ij} p_j + \eta_i}, \quad (18)$$

for appropriate definitions of  $\phi_{ij}$ . The charge  $c(r_i, \gamma_i, p_i)$  can include both the congestion charge for shared resources in the wireless network and, as we will see in Section 3.2.4, the congestion charge for resources in the wireline network and the cost of battery power at the mobile (in the uplink direction). Specific formulations for the uplink and downlink, based on the results of the previous section regarding resource usage in each direction, will be discussed in the following subsections.

The optimization in (17) involves two parameters: the rate  $r_i$  and the target bit-energy-to-noise-density ratio  $\gamma_i$ . An important result that we prove in Section 3.2.1 for the uplink, but which also holds for more general forms of the charge function  $c(\cdot)$ , is that the user optimization can be decomposed into two subproblems: one involving the selection of the optimal  $\gamma$ , which depends only on the packet success probability  $P_s(\gamma)$ , and one involving the selection of the optimal rate  $r$ , which depends on the user's utility and his charge.

Note that it is mathematically equivalent to perform the optimization of (17) over any of the two variables  $r_i, \gamma_i$ , and  $p_i$ . In WCDMA networks, however, due to multipath fading and mobility, the power to achieve a target bit-energy-to-noise-density ratio can vary significantly. Fast closed-loop power control between the base station and the mobile adjusts the transmission power to achieve the target  $\gamma_i$ . In this section we assume that the adaptation of the rate and signal quality occurs at a slower timescale compared to fast closed-loop power control. In Section 5.1 we will investigate the option of performing power control based on a more direct application of congestion pricing.

### **3.2 Uplink**

In this section we consider the uplink direction, and first assume there is a large number of mobiles, each using a small portion of the total capacity. Note, however, that the results for this simple case also hold for the more general case. The wireless resource constraint is given by (7)

$$\sum_i r_i \gamma_i < W.$$

In order to provide the right incentives for efficient use of network resources, user  $i$ 's charge should be proportion to his resource usage, which is given by the product  $r_i \gamma_i$ . Hence, in the uplink the user optimization problem (17) is

$$\begin{aligned} &\text{maximize} && U_i(r_i P_s(\gamma_i)) - \lambda r_i \gamma_i && (19) \\ &\text{over} && r_i \geq 0, \gamma_i \geq 0, \end{aligned}$$

where  $\lambda$  is the shadow price for resource  $r_i \gamma_i$ .

Note that in the above model prices do not depend on the mobile's position. This is because the uplink is interference-limited, and interference depends on the

## Congestion Pricing for Resource Control in WCDMA

received power of the signal at the base station. On the other hand, with the approaches in [3, 7, 41, 45], where charges depend on the *transmitted* power, mobile users that are far from the base station incur a higher charge, for the same rate and signal quality, compared to users that are close to the base station. On the hand, in the downlink, as we will see in Section 3.3, a mobile user's position influences his charge, since resource usage in this case is determined by the *transmitted* power at the base station.

### 3.2.1 Properties of the optimal solution

An important property which greatly simplifies the application of (19), is that the optimal  $\gamma_i^*$  of the target bit-energy-to-noise-density ratio is independent of the price  $\lambda$  and of the user's utility. This allows the decoupling of the two problems of selecting the optimal  $\gamma_i^*$  and of adjusting the transmission rate  $r_i$ . This property is stated and proved in the following proposition (the proofs for all propositions are in Appendix A).

**Proposition 1** *Let  $U_i(x_i)$  and  $P_s(\gamma_i)$  be continuously differentiable functions of the throughput  $x_i = r_i P_s(\gamma_i)$  and the target bit-energy-to-noise-density ratio  $\gamma_i$ , respectively. Also assume that  $U_i'(x_i) > 0$  for all  $x_i \geq 0$ . If there exists  $r_i^* > 0$  and  $\gamma_i^* > 0$  that achieve the maximum of (19), then  $\gamma_i^*$  is independent of the price  $\lambda$  and the utility, and satisfies*

$$P_s(\gamma_i^*) = P_s'(\gamma_i^*)\gamma_i^*. \quad (20)$$

It is important to note that Proposition 1 can be proved for more general cases where the charge has the form  $c(r\gamma)$  or  $c(rP_s(\gamma))$ . The extensions, discussed in Section 3.2.4, to the basic problem given by (19), as well as the net utility maximization problem for the downlink, discussed in Section 3.3, are of this form.

An interesting observation is that the optimal  $\gamma_i^*$ , in the case  $\gamma_i^* > 0$ , satisfying (20) also maximizes the number of bits successfully received per unit of energy [3, 7]:

$$\max_{\gamma_i > 0} \frac{r_i P_s(\gamma_i)}{p_i}, \quad (21)$$

since substituting (2) in the last equation gives

$$\max_{\gamma_i > 0} \frac{W}{I_i + \eta} \frac{P_s(\gamma_i)}{\gamma_i},$$

which is maximized for  $\gamma_i^*$  satisfying (20). This last observation indicates that the optimal  $\gamma_i^*$  that maximizes a user's net utility in the case of congestion pricing (19), where the free variables are  $r_i$  and  $\gamma_i$ , also maximizes the number of bits successfully received per unit of energy. Moreover, under the assumptions of Proposition 1, this is independent of the user's utility and the congestion price, and is achieved in a decentralized manner via pricing. Finally, we note that (20) also holds when the objective is to maximize the total throughput [33, 32].

The next proposition is related to the existence of a  $\gamma_i^* > 0$  (for simplicity we have dropped the subscript  $i$ ).

**Proposition 2** *Assume that  $P_s(\gamma)$  is continuously differentiable, and is strictly convex for  $\gamma < \gamma^0$  and strictly concave for  $\gamma > \gamma^1$ . Also assume that  $P_s(0) = 0$ . Then there exists  $\gamma^* > 0$  that satisfies (20). Moreover, if  $\gamma^0 = \gamma^1$ , then  $\gamma^*$  is unique.*

In practise, we can have  $P_s(0) > 0$ , i.e., the packet success probability does not tend to zero as  $\gamma$  tends to zero;  $\gamma = 0$ , hence  $p = 0$ , corresponds to the case where the receiver is *guessing* what the bits transmitted by the sender are. However,

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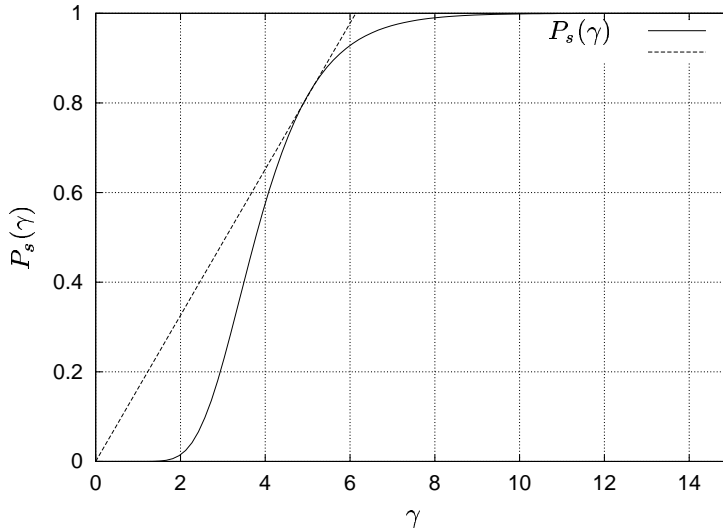


Figure 4: Packet success rate as a function of  $\gamma$  for DPSK modulation, when there is no error correction and the packet length is  $L = 60$  bits. The optimal  $\gamma^* (\approx 5)$  that achieves the maximum in (19) satisfies (20), and can be found graphically as the value of  $\gamma$  at which the line passing through the origin is tangent to  $P_s(\gamma)$ .

as we demonstrate next,  $P_s(0)$  will typically be very small, and  $\gamma^*$  satisfying (20) will exist. Further investigations with various modulation schemes, various packet lengths, and forward error correction are presented in Appendix B.1.

In the case of additive white Gaussian noise and a non-fading channel, the bit error rate for DPSK (Differential Phase Shift Keying) modulation is [37]:

$$BER(\gamma) = 0.5e^{-\gamma}.$$

From the last equation, observe that  $BER(0) > 0$ . If there is no error correction, and bit errors are independent and are all detected, then the packet success rate  $P_s(\gamma)$  is given by

$$P_s(\gamma) = (1 - BER(\gamma))^L,$$

where  $L$  is the number of bits in one packet. For packet length  $L = 60$  bits, the last equation gives  $P_s(0) = 8.710^{-19}$ .

Figure 4 shows that the packet success rate  $P_s(\gamma)$  is a sigmoid function, for which there exists a unique  $\gamma^*$  satisfying (20). The optimality of  $\gamma^*$  is shown in the following two propositions.

**Proposition 3** *Let  $\gamma^*$  be the unique value satisfying (20), and assume  $P_s''(\gamma) < 0$ . If the utility  $U(x)$  in (19) is differentiable and strictly concave and  $U'(x) > 0, \forall x > 0$ , where  $x = rP_s(\gamma)$ , then there exists a  $r^*$ , that along with the  $\gamma^*$  achieves the maximum in (19).*

**Proposition 4** *Under the conditions stated in Propositions 1 and 3, and if  $U_i(x_i)$  is increasing and strictly concave in  $x_i = r_iP_s(\gamma_i)$ , then there exists a price  $\lambda$  such that the allocations  $\{(r_i, \gamma_i)\}$  formed from the unique solutions  $(r_i, \gamma_i)$  to (19) maximize the network revenue*

$$\begin{aligned} & \text{maximize} && \sum_i \lambda r_i \gamma_i \\ & \text{over} && r_i \geq 0, \gamma_i \geq 0 \\ & \text{subject to} && \sum_i r_i \gamma_i < W, \end{aligned}$$

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and the social welfare

$$\begin{aligned}
 & \text{maximize} && \sum_i U_i(r_i P_s(\gamma_i)) \\
 & \text{over} && r_i \geq 0, \gamma_i \geq 0 \\
 & \text{subject to} && \sum_i r_i \gamma_i < W.
 \end{aligned}$$

Due to Propositions 1 and 3, the user optimization problem in (19) can be reduced to

$$\begin{aligned}
 & \text{maximize} && U_i(r_i P_s(\gamma^*)) - \lambda r_i \gamma^* \\
 & \text{over} && r_i \geq 0,
 \end{aligned} \tag{22}$$

where  $\gamma^*$  satisfies (20). In the case of a strictly concave utility, the optimal  $r_i^*$  is given by

$$r_i^* = \frac{1}{P_s(\gamma^*)} U_i'^{-1} \left( \frac{\lambda \gamma^*}{P_s(\gamma^*)} \right). \tag{23}$$

Assume now that the utility is not a strictly concave function of the rate, but has a sigmoid shape and is bounded by the line  $\xi r$ , which is tangent to the utility  $U_i(r_i P_s(\gamma^*))$  at rate  $r_i^0$ , after which the utility is strictly concave. In this case, the optimal rate is  $r_i^*$  is given by (23) if and only if

$$r_i^* \geq r_i^0.$$

If the above inequality does not hold, then the optimal rate is zero. In this case,  $\gamma_i^*$  can take any value, since both the utility and the charge is zero (equation (20) need not hold in this case). For simplicity, we will assume that  $\gamma_i^* = 0$ , when  $r_i^* = 0$ .

### **3.2.2 Procedure with explicit communication of prices**

Next we describe the procedure for applying the congestion pricing model for the up-link, described in Section 3.2, when there is direct communication of prices between the base station/radio network controller<sup>11</sup> (BS/RNC) and mobile host (MH).

1. For each MH  $i$ , the RNC selects the optimal  $\gamma_i^*$  based on (20).
2. The RNC announces the price per unit of wireless resource  $\lambda$ .
3. Each MH  $i$  selects its transmission rate  $r_i^*$  based on (22).
4. The RNC charges MH  $i$  by  $\lambda r_i \gamma_i$ .
5. The RNC adjusts price  $\lambda$  based on the load, and goes to Step 2.

In WCDMA, the procedure for selecting  $\gamma$  (target  $E_b/N_0$ ) is performed at the RNC, during outer loop power control: The BS measures the bit error rate *BER* (or the frame error rate *FER*), and sends the measurement to the RNC, which adjusts  $\gamma$  to achieve a target *BER*;  $\gamma$  is then used as the target for fast closed-loop power control, which operates between the base station and the mobiles. Hence, it is appropriate to perform the selection of the optimal  $\gamma^*$  in Step 1 at the RNC, effectively replacing the normal outer loop power control procedure. Also, note that the selection of  $\gamma^*$  can take place at the beginning of data transmission, or whenever the dependence of the packet success probability on  $\gamma$  changes, e.g., when

---

<sup>11</sup>Typically, high-level resource management procedures, such as selection of the target bit-energy-to-noise-density ratio and packet scheduling, operate at the RNC; data and control information to and from the mobile goes through the BS.

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the multipath characteristics change. The alternative to the above is to perform the selection of  $\gamma^*$  at the mobile host; such an approach, however, does not have apparent advantages and would result in higher signalling overhead, since measurements of *BER* would need to be sent to the mobile host, and in increased complexity of the mobile host.

As noted above,  $\gamma^*$  is used as the target for fast closed-loop power control between the base station and the mobile; this power control loop operates on a much faster timescale compared to the timescale over which the transmission rate is adjusted. Indeed, in WCDMA fast closed-loop power control<sup>12</sup> operates at a frequency of 1500 Hz, resulting in one power update approximately every 0.67 milliseconds. On the other hand, the rate remains constant within a single frame, whose minimum duration is 10 milliseconds. Hence, the rate control procedure described above works on top of fast closed-loop power control. Also note that, according to (2), a change in the transmission rate would require adjusting the transmission power in order to maintain the same  $\gamma^*$ .

Regarding the selection of optimal  $r_i^*$  in Step 3, recall that the spreading factor can obtain discrete values, ranging from 4 to 256 (512 in the downlink) in powers of 2, hence the transmission rate can obtain discrete values.

In Step 4, charges are proportion to the product  $r_i\gamma_i$ . The BS/RNC, assuming perfect error detection, can compute the transmission rate  $r_i$ . Also, the BS/RNC knows  $\gamma_i$ , since it has computed it (or has received it from the mobile). Hence, there is no parameter that the mobile user can falsely declare in order to reduce his charge, without reducing his level of service.

Step 5 involves adjusting the price  $\lambda$  based on some estimate of the level of congestion of wireless network resources. The specific procedure for adjusting the price is related to how prices are communicated to the mobile user. One option is to have the RNC directly announce prices; this requires a new control channel from the RNC to the MH's.

In the case of explicit price announcement, the price function is of the form  $\lambda(\rho) : [0, 1] \rightarrow [0, \infty]$ . One possible price function is the following:

$$\lambda(\rho) = \frac{\phi}{1 - \rho},$$

where  $\phi$  can be adjusted to achieve a target utilization, if a rough estimate of the demand is known. Another alternative is to have the price adjusted in fixed time intervals  $k$ , according to

$$\lambda(k + 1) = \lambda(k) + \kappa(\lambda(k))(\rho - \rho_{target}),$$

where  $\rho_{target} < 1$ , and  $\kappa(\lambda(k))$  determines the magnitude of the price change in each update.

Both of the above two alternatives require measurements of the total load  $\rho$ . One approach for measuring the total load involves direct application of  $\sum_i \alpha_i^{UL}$ , with  $\alpha_i^{UL}$  given by (4). A more efficient method is to use measurements of the total interference power  $I_{total}$  (which includes the noise), and the noise power  $\eta$ , from which the total load can be estimated using, see Section 2.1,

$$\sum_j \alpha_j^{UL} = \frac{I_{total} - \eta}{I_{total}}.$$

Advantages of the above are that only aggregate power measurements are required and CDMA's soft capacity property is implicitly handled.

<sup>12</sup>In WCDMA, fast closed-loop power control, which operates at the physical layer, is supported on dedicated channels and shared channels in both the uplink and the downlink, and on the uplink common packet channel.

3.2.3 Allocation of rates by RNC according to willingness-to-pay

An alternative to the approach discussed in the previous subsections, that involves communication of prices from the base station to the mobile hosts and rate adaptation by the mobile hosts, is to add more intelligence to the RNC, which allocates rates according to the users' declared willingness-to-pay values, Figure 5.

Such an approach is attractive for the following reasons: i) WCDMA supports negotiation of bearer service properties both at call setup, and during a call [12, p. 10]; ii) the radio network controller (RNC) already has intelligence for supporting flexible packet scheduling; iii) radio networks are single hop networks<sup>13</sup>, hence the approach we describe satisfies desirable fairness properties, namely proportional fairness [17]; iv) the approach is less demanding for mobile hosts, which do not need to adjust their rate in the (relatively fast) timescale over which the congestion price changes, but rather adjust their willingness-to-pay on a slower timescale.

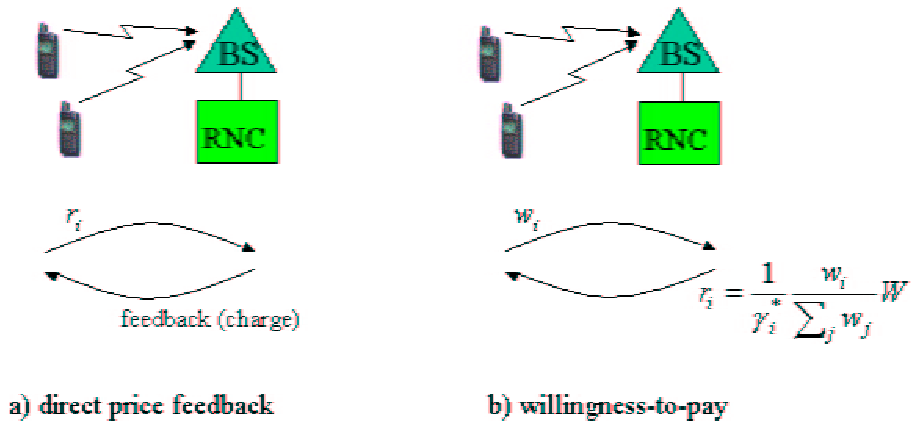


Figure 5: In the direct price communication approach (left figure), the mobile responds to price feedback by adjusting its transmission rate. On the other hand, in the willingness-to-pay approach (right figure), adjustment of the willingness-to-pay factor can be done over longer timescales. The first approach results in more efficient behavior, but requires more complex functionality in the mobile compared to the second approach.

The scheme works as follows: Mobile users signal their willingness-to-pay to the RNC, which then allocates rates in proportion to the declared willingness-to-pay. In particular, the rate for user  $i$  is given by

$$r_i = \frac{1}{\gamma_i^*} \frac{w_i}{\sum_j w_j} W,$$

where as before  $\gamma_i^*$  is given by (20). Observe from the last equation that a user's sending rate is inversely proportional to his target bit-energy-to-noise-density ratio  $\gamma_i^*$ . Also note that the rate  $r_i$  needs to be signaled from the RNC to the mobile  $i$ .

If we assume that updates of the willingness-to-pay can occur in intervals of fixed duration  $\tau$ , then the user can vary, on a slow timescale, his willingness-to-pay according to  $w_i(t) = \hat{\lambda}(t) \gamma_i^* r_i^*(t)$ , with  $r_i^*(t)$  and  $\gamma_i^*$  given by (23) and (20) respectively, and  $\hat{\lambda}(t)$  is an estimate of the price, e.g.,  $\hat{\lambda}(t) = \frac{w_i(t-\tau)}{r_i(t-\tau) \gamma_i^*}$ , with  $r_i(t-\tau)$  the rate allocated to user  $i$  at time  $t - \tau$ .

<sup>13</sup>Achieving similar fairness objectives in a multiple hop network, with rate allocation done by the routers, is more complicated.

### 3.2.4 Extensions

In this section we consider extensions to the basic model of (17). For all extensions, Propositions 1-3 hold, hence the corresponding user optimization problems can be reduced to a form similar to (22) and (20).

#### Small number of mobile hosts

If the number of mobile hosts is not large, then the resource constraint is given by (5) rather than (7). In this case, the user problem becomes

$$\begin{aligned} & \text{maximize} && U_i(r_i P_s(\gamma_i)) - \lambda \alpha_i^{\text{UL}} \\ & \text{over} && r_i \geq 0, \gamma_i \geq 0, \end{aligned} \quad (24)$$

with  $\alpha_i^{\text{UL}}$  given by (4), i.e.,

$$\alpha_i^{\text{UL}} = \frac{1}{\left(\frac{W}{r_i \gamma_i} + 1\right)}.$$

It can be shown that Propositions 1-3 still hold, hence the optimization in (24) can be performed over  $r_i$ , with  $\gamma_i$  given by (20).

#### Including cost of battery power

The cost of battery power can be included by adding an appropriate term to (19). For example, if the battery cost is linear to the power, we have

$$\begin{aligned} & \text{maximize} && U_i(r_i P_s(\gamma_i)) - \lambda r_i \gamma_i - \nu_i p_i \\ & \text{over} && r_i \geq 0, \gamma_i \geq 0, \end{aligned} \quad (25)$$

where  $p_i$  is the transmitted power and  $\nu_i$  is the cost per unit of battery power. Since, as shown in (9) or (18), the power is proportional to the product  $r_i \gamma_i$ , Propositions 1-3 still hold, hence the optimization in (25) can be performed over  $r_i$ , with  $\gamma_i$  given by (20).

Observe that the price per unit of battery power may be different for different users; this is motivated both by technological consideration, e.g., different mobiles might have different power supply capacities, and by user related constraints, e.g., depending on their location, different users might have a different ability to recharge their mobile's battery.

#### Integration with transport layer congestion control

The congestion cost associated with the fixed network can be taken into account by modifying (19) to

$$\begin{aligned} & \text{maximize} && U_i(r_i P_s(\gamma_i)) - \lambda r_i \gamma - \mu r_i P_s(\gamma_i) \\ & \text{over} && r_i \geq 0, \gamma_i \geq 0, \end{aligned} \quad (26)$$

where  $\mu$  is the price per unit of bandwidth in the fixed network. Observe that the congestion charge for the fixed network is proportional to the rate of successful data transfer over the wireless network, which is given by  $r_i P_s(\gamma)$ . As with the previous extensions, Propositions 1-3 still hold.

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### Bound on total interference produced by elastic traffic

A provider might wish to limit the total interference that elastic traffic causes to other types of traffic, such as real-time traffic, to be  $p_{max}$ . In this case, the target function in the user problem (19) remains the same, but the constraint (7) changes to

$$\sum_j g_j p_j \leq p_{max}.$$

### 3.3 Downlink

The capacity constraint in the downlink is in terms of the maximum power  $\bar{p}$  that the base station can transmit (10), hence

$$\sum_i p_i \leq \bar{p}.$$

Hence, an incentive compatible pricing scheme would be for the network to charge the mobile users in proportion to the power  $p_i$ . In this case, the user optimization problem becomes

$$\begin{aligned} & \text{maximize} && U_i(r_i(P_s(\gamma_i))) - \lambda p_i \\ & \text{over} && r_i \geq 0, \gamma_i \geq 0, \end{aligned} \quad (27)$$

where  $\lambda$  is the price per unit of power, and the variables  $r_i$ ,  $\gamma_i$ , and  $p_i$  are related through (9)

$$\gamma_i = \frac{W}{r_i} \frac{g_i p_i}{I_i + \eta_i},$$

where  $I_i = \theta_i g_i \sum_{j \neq i} p_j$  is the interference experienced at mobile  $i$ , due to signals destined to other mobiles.

In the above model, mobile users that are far from the base station incur a higher charge, for the same rate  $r$  and target bit-energy-to-noise-density ratio  $\gamma$ . As a result, users far from the base station will send at a lower transmission rate; related investigations are presented in Appendix B.2. This results in more efficient utilization of the power at the base station, since it leads to higher aggregate utility.

Note that direct use of the congestion charge in (27) to actually charge mobile users might have disadvantages, since it is an additional source for variability of prices (recall that prices are dynamic, since they depend on the level of congestion).

Rather than the power constraint (10), one can consider the constraint (16). In this case, a user is charged based on the product  $r_i \gamma_i$ , hence the user problem is

$$\begin{aligned} & \text{maximize} && U_i(r_i(P_s(\gamma_i))) - \lambda r_i \gamma_i \\ & \text{over} && r_i \geq 0, \gamma_i \geq 0. \end{aligned} \quad (28)$$

Under (28), a user's charge depends only on the performance he experiences, in terms of the transmission rate and signal quality, and is independent of his position.

Finally, note that Propositions 1-3 hold for both (27) and (28).

#### 3.3.1 Procedure with explicit communication of prices

The procedure for implementing (27), assuming explicit communication of prices from the RNC to mobile hosts, is the following:

1. Each MH  $i$  selects the target  $\gamma_i^*$  based on (20).
2. The RNC announces the price per unit of power  $\lambda$ .

### ***Congestion Pricing for Resource Control in WCDMA***

3. MH  $i$  computes  $r_i$  from (27) and signals its selection to the RNC.
4. The RNC charges MH  $i$  by  $\lambda p_i$ .
5. The RNC adjusts the price  $\lambda$  based on the load, and goes to Step 2.

Step 1 would replace the normal outer loop power control procedure operating in the MH; see [12, p. 202].

Step 3 requires that the MH has some estimate of the average values for the path gain  $g_i$ , the interference  $I_i$ , and the noise  $\eta_i$ . The latter two can be measured at the mobile host, whereas the path gain can be estimated using the received power of pilot bits in the downlink [12].

The price adjustment in Step 5 can follow the same procedures as those discussed in Section 3.2.2, with the difference that here, the total load is based on measurements of total transmitted power:

$$\rho = \frac{\sum_i \tilde{p}_i}{\bar{p}},$$

where  $\tilde{p}_i$  is the average power of the signal transmitted to mobile  $i$ .

If charges are proportion to the product  $r_i \gamma_i$ , then the above procedure is modified such that at Step 2, the selection of  $r_i$  is made based on (28), and in Step 3, the charge is  $\lambda r_i \gamma_i$ .

#### **3.3.2 Allocation of rates by RNC according to willingness-to-pay**

In such an approach, similar to the description for the uplink in Section 3.2.3, mobile users declare a willingness to pay to the RNC, which then allocates rates according to the willingness to pay of all users. As in the uplink case, the target bit-energy-to-noise-density ratio is selected based on (20). The average power  $\tilde{p}_i$  allocated to mobile  $i$  is computed from

$$\tilde{p}_i = \frac{w_i}{\sum_j w_j} \bar{p}.$$

From  $\tilde{p}_i$ , the transmission rate  $r_i$  is estimated from (9):

$$r_i = \frac{W}{\gamma_i^*} \frac{g_i}{I_i + \eta_i} \frac{w_i}{\sum_j w_j} \bar{p}.$$

Hence, the above approach requires the RNC to have knowledge of the path gain, the interference plus noise, and the target bit-energy-to-noise-density ratio. Although WCDMA supports the reporting of such parameters to the RNC [12], there is an issue of truthful declaration of the parameters from the mobiles.

The above approach differentiates mobile users based on their distance from the base station. If no such differentiation is desired, then the constraint (16) can be used, in which case the transmission rate  $r_i$  allocated to user  $i$  is given by

$$r_i = \frac{1}{\gamma_i^*} \frac{w_i}{\sum_j w_j} \frac{W \bar{p}}{\bar{\eta} + \bar{\theta} \bar{p}}.$$

In this case, the RNC does not require per-mobile measurements of the path gain and interference.

## **4 Congestion pricing for hybrid code/time division scheduling**

WCDMA supports both code division and time scheduling for packet transmission [12]. Time division scheduling has the advantage of supporting higher transfer rates

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for the same energy per transmitted bit, compared to code division scheduling, but requires synchronization and has the disadvantage of non-continuous transmission, which results in bursty traffic. Indeed, [35] shows that in a hybrid code and time division scheduling system supporting real-time (delay intolerant) and non real-time (delay tolerant) traffic, both with fixed target  $E_b/N_0$ , the aggregate transmission rate of non real-time traffic is maximized if it is scheduled so that only one non real-time source sends traffic in each time slot. Unlike time division multiplexing, code division scheduling supports continuous data transmission, but has the disadvantage of lower instantaneous bit rates.

Shared channels and the common packet channel (used in the uplink) in WCDMA typically use both time division and code division multiplexing. In the downlink, orthogonal codes are shared between many users in a time division manner, i.e., there may be many common packet channels per cell, each having a different bit rate and shared amongst many users in a time division manner. On the other hand, dedicated channels typically use code division scheduling, hence in the downlink an orthogonal code is consumed for each user of a dedicated channel. Indeed, for dedicated channels the bit rate can change during transmission, but remains constant within a single frame that has a minimum duration of 10 ms, and the orthogonal code must be allocated according to the highest bit rate. Nevertheless, the specifications do not preclude using time division scheduling for dedicated channels.

Our objective in this section is to investigate the tradeoffs between code division and time division multiplexing solely in terms of the net utility maximization problem.

First, observe that for hybrid code and time division multiplexing, the constraint on resource usage in the uplink becomes

$$\sum_i \alpha_i \varrho_i < 1,$$

where  $\alpha_i = \frac{r_i \gamma_i}{W + r_i \gamma_i}$  is the resource usage for the uplink in pure code division multiplexing systems, and  $\varrho_i$  is the percentage of time slots in which user  $i$  sends traffic.

Next, we discuss a utility model for elastic users which value, in addition to the average throughput, whether they can continuously transmit data. For the latter, we consider the expression

$$U_{cont,i}(\varrho_i),$$

where  $\varrho_i$  is percentage of time slots in which user  $i$  sends data.

The overall utility for a user that values both the average throughput and how continuous his transmission is, can be taken to be

$$U_i(\varrho_i r_i P_{s,i}(\gamma_i)) + U_{cont,i}(\varrho_i),$$

hence the user's net utility maximization problem is

$$\begin{aligned} \text{maximize} \quad & U_i(\varrho_i r_i P_{s,i}(\gamma_i)) + U_{cont,i}(\varrho_i) - \lambda \alpha_i \varrho_i \\ \text{over} \quad & r_i \geq 0, \gamma_i \geq 0, \varrho_i \geq 0, \end{aligned} \quad (29)$$

Taking the partial derivatives, with respect to  $r, \gamma, \varrho$ , of the objective function in (29) and equating them with zero gives:

$$U_i'(\varrho_i^* r_i^* P_{s,i}(\gamma_i^*)) P_{s,i}(\gamma_i^*) = \lambda \frac{W \gamma_i^*}{(W + r_i^* \gamma_i^*)^2}, \quad (30)$$

$$U_i'(\varrho_i^* r_i^* P_{s,i}(\gamma_i^*)) P_{s,i}'(\gamma_i^*) = \lambda \frac{W}{(W + r_i^* \gamma_i^*)^2}, \quad (31)$$

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$$U'_i(\varrho_i^* r_i^* P_{s,i}(\gamma_i^*)) r_i^* P_{s,i}(\gamma_i^*) + U'_{cont,i}(\varrho_i^*) = \lambda \frac{r_i^* \gamma_i^*}{W + r_i^* \gamma_i^*}, \quad (32)$$

From (30) and (31) we find that the optimal  $\gamma_i^*$  satisfies (20), i.e., Proposition 1 holds. Moreover, from (32) and (30) we have

$$\begin{aligned} \lambda \frac{W r_i^* \gamma_i^*}{(W + r_i^* \gamma_i^*)^2} + U'_{cont,i}(\varrho_i^*) &= \lambda \frac{r_i^* \gamma_i^*}{W + r_i^* \gamma_i^*} \Rightarrow \\ U'_{cont,i}(\varrho_i^*) &= \lambda \frac{(r_i^* \gamma_i^*)^2}{(W + r_i^* \gamma_i^*)^2}. \end{aligned} \quad (33)$$

If  $U_{cont,i}(\varrho)$  is increasing and strictly concave, then from (33) we have that a smaller congestion price  $\lambda$  results in a larger  $\varrho_i^*$ . Moreover, for a larger number of mobile users, the optimal value  $\varrho_i^*$  is larger. Indeed, if we had assumed  $\alpha_i \approx r_i \gamma_i / W$ , which becomes accurate when there is a large number of mobile users, then  $U'_{cont,i}(\varrho_i) \approx 0$ , hence the value of  $\varrho_i$  would be selected so as to maximize the utility  $U_{cont,i}(\varrho_i)$ .

A more detailed analysis to that presented in this section would include a minimum transmission rate, which is required for synchronization in time division scheduling; e.g., see [35].

## 5 Congestion pricing for rate-inelastic traffic

In this section we consider rate-inelastic traffic, which has minimum rate requirements, but can adapt its target bit-energy-to-noise-density ratio. Such applications include, e.g., streaming video/audio, which can have a fixed transmission rate, but whose quality, as perceived by users, depends on the frame error rate; the latter depends on the signal quality, which is determined by the target bit-energy-to-noise-density ratio.

A possible expression for the utility of rate-inelastic traffic is

$$U_{in}(r, \gamma) = U_r(r) U_q(\gamma),$$

where  $U_r(r)$ , due to the inelasticity in terms of the rate, is a step function and  $U_q(\gamma)$  can be an increasing concave or a sigmoid function, Figure 6; a utility with a sigmoid shape is able to capture minimum requirements in terms of  $\gamma$ , and can be justified by the shape of the packet success rate as a function of  $\gamma$ , Figure 4.

### Uplink

In the uplink, as discussed in Section 3.2, the congestion charge for user  $i$  is proportion to the product  $r_i \gamma_i$ , hence the user problem involves the following maximization (without loss of generality, we assume that  $U_r(r_{min}) = 1$ ):

$$\begin{aligned} \text{maximize} \quad & U_{q,i}(\gamma_i) - \lambda r_{min,i} \gamma_i \\ \text{over} \quad & \gamma_i \geq 0. \end{aligned} \quad (34)$$

The optimal  $\gamma_i^*$  for achieving the maximum in (34) satisfies

$$U'_{q,i}(\gamma_i^*) = \lambda r_{min,i}. \quad (35)$$

Note that if  $U_{q,i}(\gamma_i)$  is an increasing and strictly concave function of  $\gamma_i$ , then an optimal  $\gamma_i^*$  exists and is unique.

On the other hand, if  $U_{q,i}(\gamma_i)$  has a sigmoid shape, Figure 6, and is bounded by the line  $\xi \gamma_i$ , which is tangent to the utility at  $\gamma_i^0$ , after which the utility is strictly concave, then the user will enter the system if and only if  $\gamma_i^* \geq \gamma_i^0$ , in which case the optimal bit-energy-to-noise-density ratio is  $\gamma_i^*$ .

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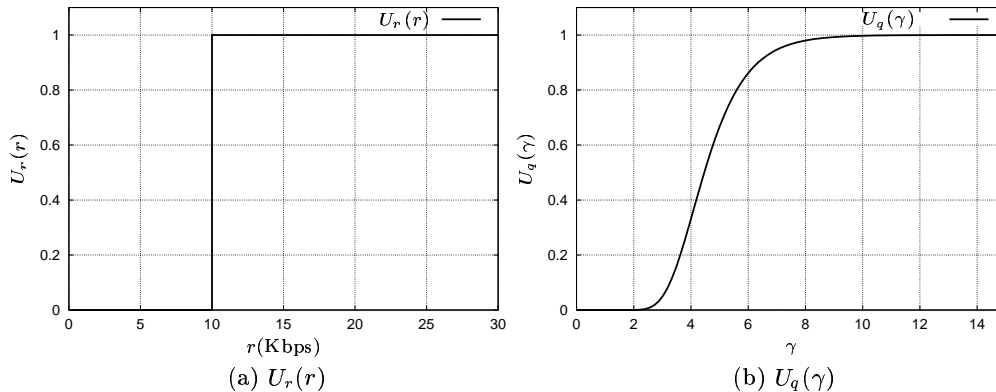


Figure 6: Utility for rate  $r$  and target bit-energy-to-noise-density ratio  $\gamma$ .

### Downlink

In the downlink, as discussed in Section 3.3, charges are proportion to the transmission power. Hence, the user objective is to maximize the expression

$$U_{q,i}(\gamma_i) - \lambda p_i,$$

where  $r_i$ ,  $\gamma_i$ , and  $p_i$  are related through (9). The above optimization can be performed over the target bit-energy-to-noise-density ratio  $\gamma_i$ ; such an approach would require some estimates of the average value for the path gain, the interference, and the noise, which together with the rate  $r_{min,i}$  and signal quality  $\gamma_i$  determine the average power, hence the average charge. The value  $\gamma_i^*$  that maximizes the net utility is then handed to fast closed-loop power control, which adjusts the power in order to achieve this target.

Alternatively, the optimization can be performed over the transmitted power. In this case, power adjustments on a fast timescale are based on congestion pricing. This is unlike the models we have considered up to now, where the adaptation of the rate and bit-energy-to-noise-density ratio occurred on a timescale slower than the timescale of fast closed-loop power control. We investigate this approach further in the following subsection.

### 5.1 Efficient and robust power control via congestion pricing

The models we considered up to now involved optimization over the rate and signal quality that occurred in timescales slower than the timescales of closed-loop power control, Figure 7.

In this subsection we consider the optimization of the net utility over the power, and based on this model we consider a power control algorithm, where power updates depend on congestion charges. Important properties of the algorithm are the following: i) it is fair in that a mobile user experiences a congestion charge that depends on the amount of resource he uses, ii) it is efficient in terms of optimizing the social welfare of the wireless network, and iii) it is robust in terms of handling congestion. On the other hand, traditional power control algorithms, such as the standard power control algorithm in [46], converge only if they are feasible, i.e., if there exists a set of power values for which the target  $E_b/N_0$  of all mobile users is satisfied.

In the downlink<sup>14</sup> the user optimization problem, in the case of rate inelastic

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<sup>14</sup>In the uplink, where the user optimization problem is given by (34), prices depend on the

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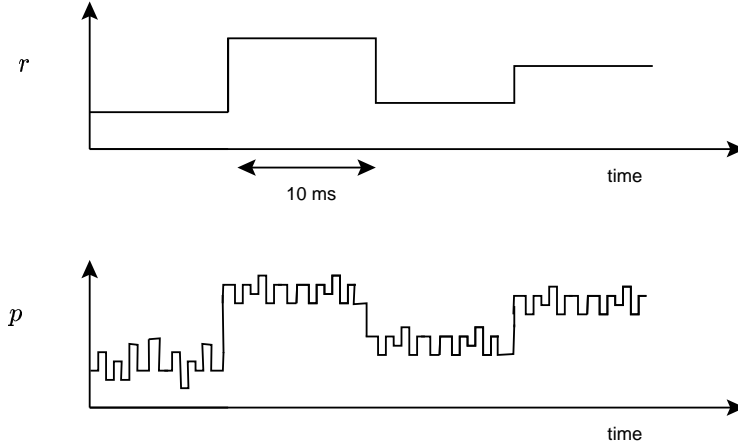


Figure 7: Timescales of rate and power adaptation. Rates remain constant within a single frame, that has a minimum duration of 10 ms. Power control operates at 1500 Hz, hence 15 power updates are performed within a single frame.

traffic, can be written as

$$\begin{aligned} & \text{maximize} && U_{q,i}(\gamma_i) - \lambda p_i \\ & \text{over} && p_i \geq 0, \end{aligned} \quad (36)$$

where  $r_i$ ,  $\gamma_i$ , and  $p_i$  are related through (9), i.e.,

$$\gamma_i = \frac{W}{r_i} \frac{g_i p_i}{\theta_i g_i \sum_j p_j + \eta_i}. \quad (37)$$

Consider the case where the utility is logarithmic, i.e.,  $U_{q,i}(\gamma_i) = w_i \log(\gamma_i)$ , where  $w_i$  is a weight parameter. Due to the linear dependence between  $\gamma_i$  and  $p_i$  in (37), the two problems

$$\begin{aligned} & \text{maximize} && w_i \log(\gamma_i) - \lambda p_i \\ & \text{over} && p_i \geq 0 \end{aligned}$$

and

$$\begin{aligned} & \text{maximize} && w_i \log(p_i) - \lambda p_i \\ & \text{over} && p_i \geq 0, \end{aligned}$$

which differ in the argument of the logarithmic function, have the same optimum, given by

$$w_i = \lambda p_i. \quad (38)$$

Next we present a power update algorithm based on (38), which has a form identical to the rate control algorithm described in [19, 18]. Let  $C(y)$  be the cost incurred at the wireless resource when its total load is  $y$ . Assume that  $C(y)$  is differentiable and

$$\frac{dC(y)}{dy} = L(y). \quad (39)$$

Consider the following system of differential equations for updating the power

$$\frac{dp_i(t)}{dt} = \kappa_i (w_i - \lambda(t)p_i(t)), \quad (40)$$

transmission rate (which is fixed) and the bit-energy-to-noise-density ratio; there is no apparent advantage of adjusting the latter in a timescale as fast as that of closed-loop power control.

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for  $i \in I$ ,  $I$  being the set of mobile users,  $\kappa_i$  is a gain factor controlling the speed of convergence, and

$$\lambda(t) = L \left( \sum_i p_i \right). \quad (41)$$

The system of differential equations (39)-(41) is a special case of the system considered in [19, 18], where it is proved that the system has a unique stable point to which all trajectories converge.

The above discussion considered a logarithmic utility. For a general form of the utility, the user can slowly vary the willingness-to-pay in (40) so as to achieve

$$w_i(t) = \gamma_i U'_i(\gamma_i).$$

A fundamental difference between the power control algorithm presented in this section and the normal distributed power control algorithms encountered in the literature is that the latter increase the power when the path loss or interference increases. On the other hand, this is not the case with a power control algorithm based on utility and congestion pricing; power will increase only if the increase of bit-energy-to-noise-density ratio results in an increase of the net utility.

## **6 Related work**

In this section we present a brief review of related work, identifying the main differences with the work presented in this report.

The authors of [7], also see [39, 3] and more recent work in [41, 40], consider a utility which is interpreted as the number of information bits transmitted per unit of energy. The utility has the property that for a bit-energy-to-noise-density ratio  $E_b/N_0$  larger than some value, the utility decreases with the increase of  $E_b/N_0$ . It is shown that for the non-cooperative game where mobile users adjust their power to maximize their utility, there exists a unique Nash equilibrium. In the equilibrium all users achieve the same bit-energy-to-noise-density ratio  $E_b/N_0$ . Moreover, the latter satisfies the same equation as the one that the optimal target bit-energy-to-noise-density ratio that maximizes the net utility in our framework satisfies.

The resulting Nash equilibrium, however, is inefficient. With the introduction of prices that are proportional to the power, they achieve Pareto improvements, compared to the case of no pricing; nevertheless, even with the introduction of such a pricing scheme, the social welfare optimum is not achieved [39, 41]. However, for a Nash equilibrium to exist all users must always transmit with some minimum power [41]. Finally, in [41] an algorithm for adjusting the price per unit of power is proposed which requires knowledge of the changes of the utility of users, information that needs to be communicated to the base station. The formulation does not capture the cost of congestion of wireless network resources. Other problems include the incentives for users not to declare the true value of the power with which they are transmitting, since they will be charged for it, and of the change of their utility; see also the discussion in [28, 27].

The authors of [43] consider a utility that is a function of the transmission rate, and investigate the problem of maximizing the sum of all utilities in the forward link, under constraints on the total transmission power of a base station, and for each user minimum rate constraints, which translate to maximum delay constraints, and maximum error rate constraints. The solution involves local by each mobile selection of the minimum bit-energy-to-noise-density ratio such that the maximum error rate constraint is met and the centralized, by the base station, selection of optimal rates; hence, the base station would need to know the utility of every user. The approach proposed in Section 3.3, and given by (27), considers a similar objective, namely

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that of maximizing the sum of all utilities but with the utility being a function of the actual throughput (transmission rate multiplied by the success probability), rather than the transmitted rate. Moreover, our approach uses a decentralized scheme, based on prices, to achieve the objective.

The authors of [45] consider a utility, having a sigmoid shape, that is a function of the bit-energy-to-noise-density ratio, and formulate a utility-based distributed power control algorithm where each user seeks to maximize his net utility, i.e., the difference his utility minus the cost of power, which is taken to be a linear function of the power. The authors indicate that price adjustment can be used to control resource usage, without however relating this to constraints on the wireless resources. They propose a function where prices are proportional to the interference a mobile host experiences, hence the price per unit of power can be different for different mobile hosts. The work considers only the downlink, indicating that the uplink could be handled similarly. If, however, in the uplink the transmit power is charged, then similar to the approach of the previous paper, this does not discourage mobile hosts from declaring a power different from their true power or not update the price per unit of power according to the above.

The authors of [25] consider downlink resource allocation in CDMA networks based on pricing. The user utility is a step function of the bit-energy-to-noise-density ratio, and the price each mobile is charged with contains a constant term (price per code) and a term linear in the transmitted power from the base station. It is also assumed that each base station is charged by some amount that is proportional to the total power with which it is transmitting; this charge accounts for the interference it is causing to neighboring cells. They then go on to investigate the problem of maximizing the total utility minus the base station's power charge and of the total revenue (payment received from mobile hosts) minus the base station's power charge.

The authors of [16] consider a utility that is a monotonically increasing concave function of the bit-energy-to-noise-density ratio and a monotonically decreasing concave function of the mobile host (transmitter) power. Each mobile adjusts its sending power (the rate is considered fixed) so as to maximize its utility. Under some assumption regarding the utility, a Nash (non-cooperative) equilibrium exists.

The work in [26] discusses in a qualitative manner the concept of utility, specifically as a function of bandwidth and bit error rate, at various levels of CDMA system (user, individual cell, and whole system). The concept of utility in wireless network is also considered in [23, 1, 24], while [8] considers service differentiation based on elasticity in terms of the rate for different class, which determines how rates are adjusted in periods of congestion of wireless resources.

The authors of [2] consider a cost function with two components, one linear in the transmitted power and the other linear in the target  $E_b/N_0$ . Using this cost function they formulate problems that take into account the deadline requirements of each packet, and adapt the target  $E_b/N_0$  based on these deadlines; it is assumed that the network-level QoS requirements translate to such deadline requirements.

Another line of research has considered the problem of maximizing the aggregate throughput in CDMA systems. The work in [38] considers the problems of minimizing the aggregate power and of maximizing the sum of rates, given target  $E_b/N_0$  values and constraints on the maximum power and minimum rate. Problems similar to the latter are addressed in [48, 22, 44, 15]. The authors of [35] show that in hybrid CDMA/TDMA systems supporting real-time (delay intolerant) and non real-time (delay tolerant) traffic, both with fixed target  $E_b/N_0$  values, the sum of rates of the non real-time traffic is maximized if it is scheduled so that only one non real-time source sends traffic in each time slot.

Whereas the previous work considered fixed  $E_b/N_0$  values, [13, 33] consider the case where both target  $E_b/N_0$  and the rate (equivalently, the spreading gain) can

be controlled. [13] considers a system with two classes, for data and voice, and investigates the problem of assigning powers processing gains to each class in order to optimize performance in terms of average delay for data and bit error rate for voice; it is assumed that traffic for both types follows a Poisson process. [33] investigates the maximization of throughput (taking into account losses in the wireless network) and shows that the optimal spreading gain is inversely proportional to the multiple access interference and that the optimal retransmission probability for data traffic is one; a distributed scheme for achieving the previous is investigate in [31]. Related work in [32] additionally considered a constraint on the total received power at the base station from mobile hosts with data traffic, and that the transmission power is allocated to mobile hosts with data traffic in decreasing order of their path gains.

## 7 Conclusions and further work

In this report we have investigated the application of congestion pricing for resource control in WCDMA networks. The framework presented can capture the congestion charge for resources in both wireless and wireline networks, and the cost of battery power consumption in mobile hosts. For elastic traffic, we have shown that the selection of the optimal rate and signal quality, the latter expressed in terms of the target bit-energy-to-noise-density ratio, can be decoupled. This result enables the integration of rate adaptation at the CDMA layer, with congestion control at the transport layer, while the selection of the target signal quality can remain at the CDMA layer, replacing traditional outer loop power control. The model considers adjusting the shadow price, which is communicated by the Radio Network Controller (RNC) to the mobile hosts, based on some estimate of the level of congestion. An interesting alternative to the direct announcement of prices is to use Explicit Congestion Notification (ECN) [36], which has recently been approved as an IETF proposed standard. Indeed, the use of ECN marking for conveying congestion prices has been proposed for fixed networks in [4, 21, 18, 5], whereas ECN marking for improving the performance of TCP over wireless networks has been proposed in [30], and for 3G wireless networks in [14].

Extensions to the basic model include hybrid code and time division scheduling, and rate-inelastic/quality-elastic traffic. Based on the latter model, we presented an approach for efficient and robust power control in the downlink based on congestion pricing.

Two issues related to the application of the models and procedures discussed in this report are the following: The first is related to the transmission rate in WCDMA, which can take discrete values. When congestion control is performed at the transport layer, how is the spreading factor, which determines the transmission rate, selected at the CDMA layer? The second issue is related to the downlink. We have assumed that there is a maximum amount of power that can be used for data transmission. This assumes that the amount of power used by control channels is fixed. In WCDMA, however, the power used for control channels can be some percentage of the power used in the data channels [12, p. 245-247]. How can this be taken into account in the models and procedures proposed for the downlink?

The objective of our work was to investigate the application of congestion pricing as a means for *resource control*. The actual charges and tariffs that end-users face may differ from the congestion charge, and will be influenced by factors such as competition, market structure and segmentation.

Some of the directions for further work include the following:

- Consideration of more complex scenarios with multiple cells, and the presence of soft and hard handoffs. Problems that arise in such cases include the

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coverage of each cell, the placement of base stations, etc. The application of an economic model in a multiple cell data network is investigated in [40].

- We have considered resource constraints solely in terms of average quantities, which correspond to guarantees in an average sense. A possible extension is to consider probabilistic guarantees. In this case, the burstiness of the traffic, hence of the transmitted power, needs to be taken into account. Work in this direction includes [29, 42].
- Investigation of the transient and the convergence behavior of procedures and algorithms based on congestion pricing, in a dynamic environment with the presence of mobility, variable shadowing, fast (multipath) fading, etc. A related issue is the performance of transport layer congestion control algorithms, such TCP, and its interworking with the RNC marking algorithms proposed in this report. Issues related to the performance of TCP in wireless networks are discussed in [30, 34, 14].
- We have considered the case of continuous transfer of an unlimited amount of data. In the case where there is a fixed amount of data, as in a file transfer, the utility is typically a function of the total transfer duration [20]. In mobile wireless networks, additional factors not present in fixed wireline networks, that can be taken into account in file transfer algorithms, are the position and speed of the mobile.

In addition to the above, resource control in other wireless technologies is also of particular interest. For example, 802.11 wireless local area networks will play an important role in future wireless communications. The problem of resource control in 802.11 differs from that in WCDMA systems, and cellular systems in general, in that 802.11 is an ad-hoc network where there is no device that plays the role of the radio network controller, and peer-to-peer communication between all mobile hosts is allowed.

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## A Mathematical proofs

**Proposition 1** Let  $U_i(x_i)$  and  $P_s(\gamma_i)$  be continuously differentiable functions of the throughput  $x_i = r_i P_s(\gamma_i)$  and the target bit-energy-to-noise-density ratio  $\gamma_i$ , respectively. Also assume that  $U'_i(x_i) > 0$  for all  $x_i \geq 0$ . If there exists  $r_i^* > 0$  and  $\gamma_i^* > 0$  that achieve the maximum of (19), then  $\gamma_i^*$  is independent of the price  $\lambda$  and the utility, and satisfies

$$P_s(\gamma_i^*) = P'_s(\gamma_i^*)\gamma_i^*. \quad (42)$$

*Proof:* At the optimal, the partial derivatives of (19) with respect to  $r_i$  and  $\gamma_i$  are zero, hence

$$\begin{aligned} \frac{\vartheta U_i(r_i P_s(\gamma_i^*))}{\vartheta r_i} &= \lambda \gamma_i^* \Rightarrow \\ U'_i(x_i^*) \frac{\vartheta(r_i P_s(\gamma_i^*))}{\vartheta r_i} &= \lambda \gamma_i^* \Rightarrow \\ U'_i(x_i^*) P_s(\gamma_i^*) &= \lambda \gamma_i^* \end{aligned} \quad (43)$$

and

$$\begin{aligned} \frac{\vartheta U_i(r_i^* P_s(\gamma_i))}{\vartheta \gamma_i} &= \lambda r_i^* \Rightarrow \\ U'_i(x_i^*) \frac{\vartheta(r_i^* P_s(\gamma_i))}{\vartheta \gamma_i} &= \lambda r_i^* \stackrel{r_i^* > 0}{\Rightarrow} \\ U'_i(x_i^*) P'_s(\gamma_i^*) &= \lambda \end{aligned} \quad (44)$$

From (43) and (44) we get (42). This completes the proof.  $\square$

**Proposition 2** Assume that  $P_s(\gamma)$  is continuously differentiable, and is strictly convex for  $\gamma < \gamma^0$  and strictly concave for  $\gamma > \gamma^1$ . Also assume that  $P_s(0) = 0$ . Then there exists  $\gamma^* > 0$  that satisfies (42). Moreover, if  $\gamma^0 = \gamma^1$ , then  $\gamma^*$  is unique.

*Proof:* Because  $P_s(\gamma)$  is concave for  $\gamma > \gamma^2$  and  $P_s(0) = 0$ , there exists a function  $f(\gamma) = a\gamma$  such that

$$P_s(\gamma) \leq a\gamma \quad \forall \gamma > 0.$$

Because  $P_s(\gamma)$  is initially convex, the equality in the last equation is satisfied for some  $\gamma^* > 0$ . Since  $P_s(\gamma)$  is continuously differentiable, (42) will hold for this  $\gamma^*$ .

Finally, if  $P_s(\gamma)$  is initially strictly convex and then strictly concave, then the  $\gamma^* > 0$  must be unique.  $\square$

**Proposition 3** Let  $\gamma^*$  be the unique value satisfying (42), and assume  $P''_s(\gamma) < 0$ . If the utility  $U(x)$  in (19) is differentiable and strictly concave and  $U'(x) > 0, \forall x > 0$ , where  $x = r P_s(\gamma)$ , then there exists a  $r^*$ , that along with the  $\gamma^*$  achieves the maximum in (19).

*Proof:* Due to the strict concavity of  $U(x)$ , there exists a unique  $r^*$  that achieves the the maximum of (19); this  $r^*$  satisfies (43):

$$U'(r^* P_s(\gamma^*)) P_s(\gamma^*) = \lambda \gamma^*. \quad (45)$$

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The first partial derivative with respect to  $\gamma$  of the objective function in (19) is

$$\frac{\vartheta U(r^* P_s(\gamma))}{\vartheta \gamma} - \lambda r^* = U'(r^* P_s(\gamma^*)) P_s'(\gamma^*) - \lambda r^* ,$$

which due to (45) and (42) is zero. The second partial derivative is

$$U''(r^* P_s(\gamma^*)) r^* P_s'(\gamma^*) P_s'(\gamma^*) + U'(r^* P_s(\gamma^*)) P_s''(\gamma^*) ,$$

which is less than zero since  $U'(x) > 0$ ,  $P_s'(\gamma) > 0$ ,  $U''(x) < 0$  and  $P_s''(\gamma^*) < 0$ .

Hence, the unique  $\gamma^*$  achieves the maximum of (19), since the first partial derivative is zero and the second is negative. □

**Proposition 4** *Under the conditions stated in Propositions 1 and 3, and if  $U_i(x_i)$  is increasing and strictly concave in  $x_i = r_i P_s(\gamma_i)$ , then there exists a price  $\lambda$  such that the allocations  $\{(r_i, \gamma_i)\}$  formed from the unique solutions  $(r_i, \gamma_i)$  to (19) maximize the network revenue*

$$\begin{aligned} & \text{maximize} && \sum_i \lambda r_i \gamma_i \\ & \text{over} && r_i, \gamma_i \\ & \text{subject to} && \sum_i r_i \gamma_i < W , \end{aligned}$$

and the social welfare

$$\begin{aligned} & \text{maximize} && \sum_i U_i(r_i P_s(\gamma_i)) \\ & \text{over} && r_i, \gamma_i \\ & \text{subject to} && \sum_i r_i \gamma_i < W . \end{aligned}$$

*Proof:* The above is a consequence of Propositions 1 and 3, together with Theorem 1 in [17]. □

## B Numerical investigations

### B.1 Packet success rate and net utility maximization

In this section we first investigate the packet success rate  $P_s(\gamma)$  for different modulation schemes, and its dependence on the packet length and forward error correction. Then, we present investigations related to the optimization in (19).

When there is no error correction, and bit errors are independent and are all detected, the packet success rate is given by

$$P_s(\gamma) = (1 - BER(\gamma))^L ,$$

where  $L$  is the number of bits in one packet.

When up to  $k$  bits errors are correctable, the packet success probability is given by

$$P_s(\gamma) = \sum_{j=0}^k \binom{L}{j} BER(\gamma)^j (1 - BER(\gamma))^{L-j} .$$

*Congestion Pricing for Resource Control in WCDMA*

Table 1: Bit error rate for different modulation schemes:  $Q(x) = \frac{1}{2}\text{erfc}(\frac{x}{\sqrt{2}})$ , where  $\text{erfc}$  is the complementary error function.

modulation	BER
BPSK (Binary Phase Shift Keying)	$Q(\sqrt{2\gamma})$
QPSK (Quadrature Phase Shift Keying)	$Q(\sqrt{2\gamma})$
DPSK (Differential Phase Shift Keying)	$\frac{1}{2}e^{-\gamma}$
NC-FSK (Non Coherent Frequency Shift Keying)	$\frac{1}{2}e^{-\frac{\gamma}{2}}$

The packet success rate for different modulation schemes, forward error correction, and packet lengths is shown in Figure 8. The modulation schemes that we consider, and the dependence of the bit error probability on the bit-energy-to-noise-density ratio is shown in Table 1.

Figure 8 shows that for all the modulation schemes, and for the parameters considered, there exists a  $\gamma^*$  satisfying (20). Moreover, since  $P_s''(\gamma^*) < 0$ , this  $\gamma^*$  achieves the maximum in (19).

Next we consider the utility  $U(x) = 1 - e^{-bx}$ , with  $b = 0.1$ . Figure 9 shows the net utility (utility minus charge), as a function of the rate  $r$  and  $\gamma$ , and Figure 10 shows the net utility as a function of  $r$ , for fixed  $\gamma$ . Finally, Figure 11 shows the net utility as a function of  $r$  for  $\gamma^* \approx 5$ , which satisfies (20) and can be determined graphically from Figure 4, and for  $\gamma^* \pm 1.5$ . This figure illustrates that the optimization in (19) is achieved for  $\gamma^*$ .

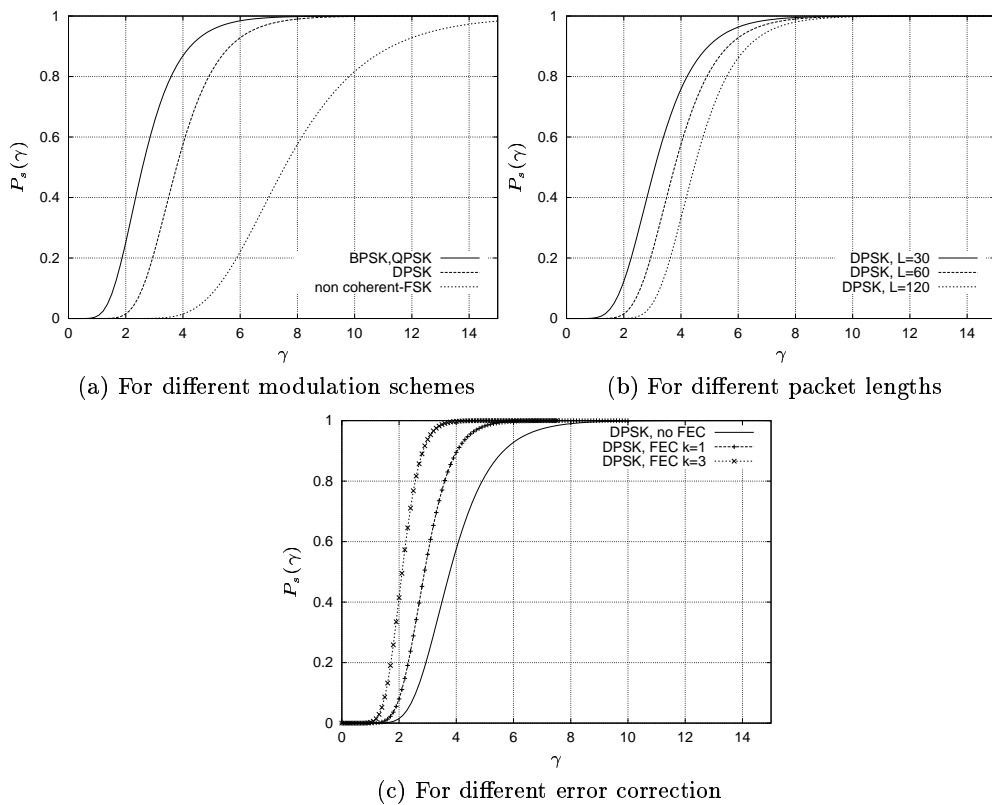


Figure 8: Packet success probability  $P_s(\gamma)$  for different modulation schemes, packet lengths, and in the presence of forward error correction. Observe that in all cases, the shape is sigmoid, and there exists a  $\gamma^*$  satisfying (20).

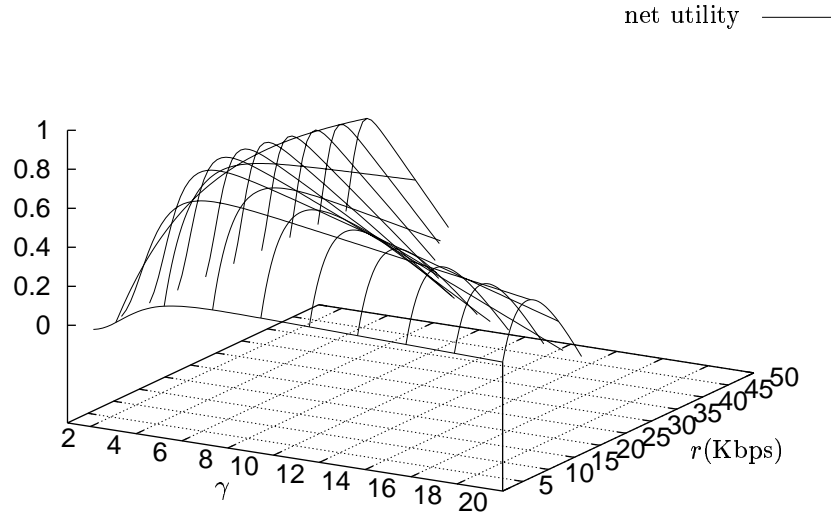


Figure 9: Net utility (benefit) as a function of rate  $r$  and target bit-energy-to-noise-density ratio  $\gamma$ . In general, the optimization of the net utility is over the two variables  $r, \gamma$ . Fortunately, this problem can be decoupled into two separate and simpler problems, Section 3.2.1: the first involves selecting the optimal  $\gamma$  (this problem is independent of the price and the utility), and the second involves adjusting the rate  $r$ .

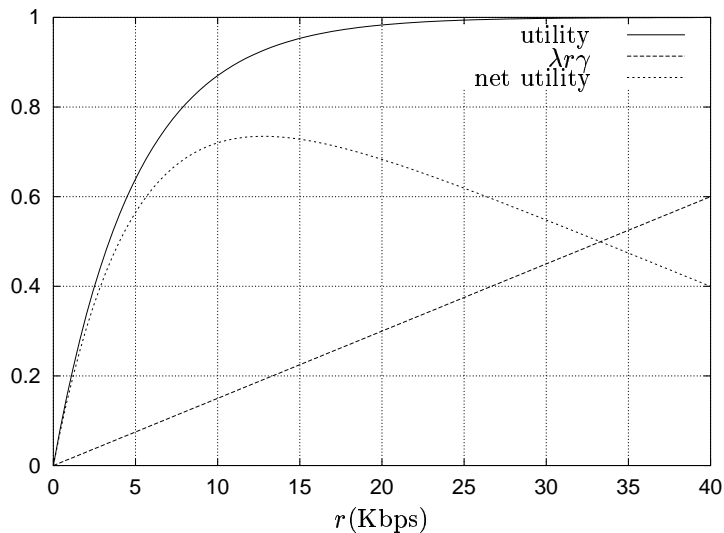


Figure 10: Utility and net utility (benefit) as a function of rate  $r$ . The maximum net utility is achieved for the rate at which the tangent to the utility has the same slope with the straight line giving the charge.

## Congestion Pricing for Resource Control in WCDMA

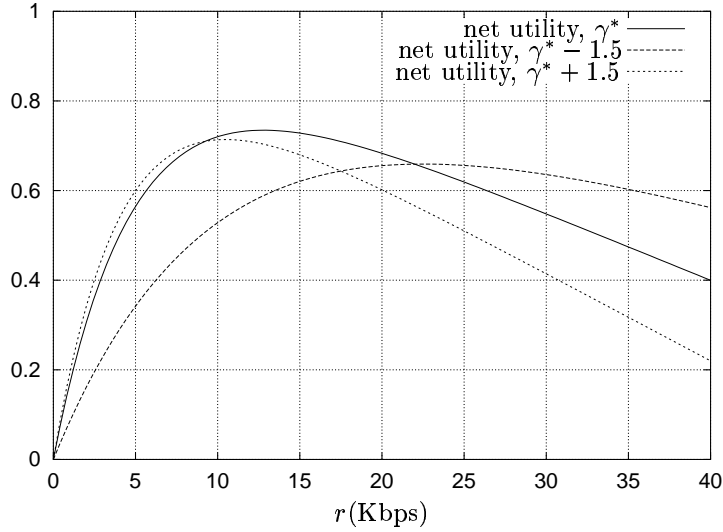


Figure 11: Net utility as a function of rate  $r$ , for  $\gamma^*$  and  $\gamma^* \pm 1.5$  ( $\gamma^* \approx 5$ ). The figure illustrates graphically that the maximum net utility is achieved for  $\gamma^*$  satisfying (20), which can be determined graphically from Figure 4.

## B.2 Comparison of uplink and downlink congestion pricing models

In this section we present numerical investigations that demonstrate the applications and differences when congestion pricing is applied to the uplink and the downlink of a CDMA network.

The rate for the uplink is computed from (19) and for the downlink from (27). In both cases,  $\gamma$  is determined from (20). The propagation model is one which corresponds to an urban environment, and is given by (46).

We consider two types of utility curves: one concave, given by  $U(x) = 1 - e^{-0.4x}$ , and one that has a sigmoid shape; in the latter case, the utility is bounded by  $\xi r$ , which is tangent to the utility  $U(rP_s(\gamma^*))$ , where  $\gamma^*$  is given by (20), at rate  $r^0 = 5.632$  Kbps, after which the utility is strictly concave and is given by  $U(x) = 1 - e^{-0.4(x-2)}$ . The other parameters are shown in Table 2.

Table 2: Parameters for the uplink-downlink investigation.  $d$  is distance in Km.

parameter	function	values
noise, $\eta$	-	$10^{-13}$ Watt
total BS power, $\bar{p}$	-	12.5 Watt
power load	-	80%
path gain, $g(d)$	$kd^{-u}$	$u = 3.52,$ $k_{urb} = 1.8197 \cdot 10^{-14},$
orthogonality factor, $\theta$	-	0.1
$BER(\gamma)$ (DPSK)	$0.5e^{-\gamma}$	
number of bits per pkt, $L$		60
$\gamma^*$ , from (20) and Fig. 4		5
Utility, concave	$1 - e^{-bx}$	$b = 0.4$
Utility, sigmoid	$1 - e^{-b(x-x_0)}, x \geq 5.632P_s(\gamma^*)$ convex, $x < 5.632P_s(\gamma^*)$	$b = 0.4, x_0 = 2$

## Congestion Pricing for Resource Control in WCDMA

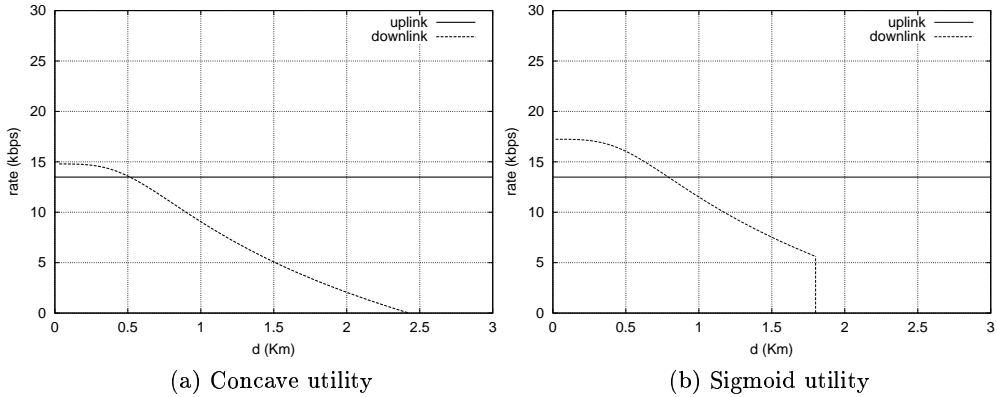


Figure 12: Rate as a function of distance. In the uplink, the rate is independent of the distance, since charges are independent of the mobile's position. On the other hand, in the downlink the rate decreases with the distance, because charges are based on the transmitted power, hence are influenced by the path gain. For a sigmoid utility, in the downlink the rate drops to zero at the distance where the net utility is negative for any positive rate.

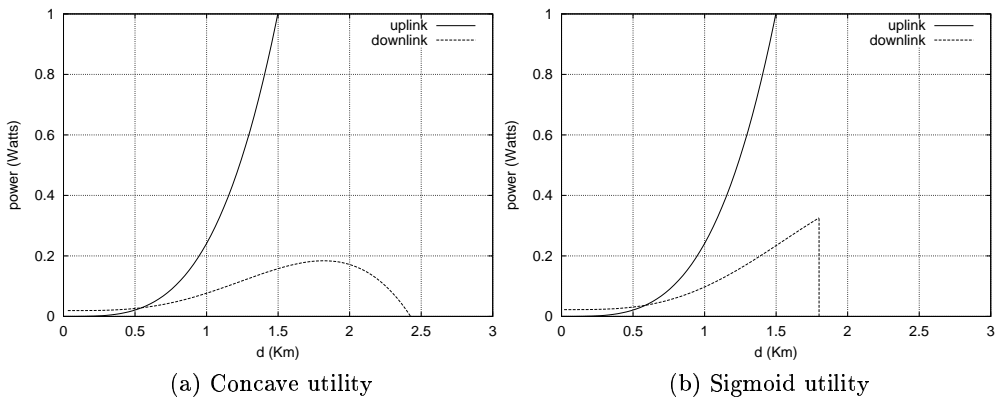


Figure 13: Power as a function of distance. In the uplink, charges are independent of the mobile's position and its power, hence the power continuously increases with the distance in order to maintain a constant signal quality. In the downlink, the power initially increases with the distance, since doing so increases the net benefit, and then decreases because the path loss increases fast with the distance, hence the necessary power and charge increases fast. For a sigmoid utility, in the downlink the rate drops to zero at some distance, where the net utility is negative for any positive rate.

## Congestion Pricing for Resource Control in WCDMA

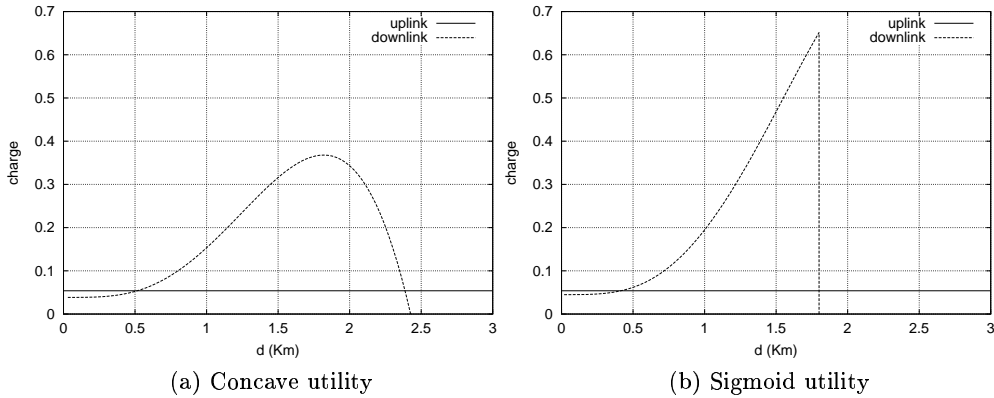


Figure 14: Charge as a function of distance. In the uplink, the charge is independent of the distance. In the downlink, the charge follows a shape similar to the power, Figure 13, since the charge is proportional to the power.

### B.3 Tradeoff between coverage and social welfare

As discussed in Section 2.1.2, when there are constraints on the power each mobile can transmit, the coverage will be limited by the mobile experiencing the largest path loss. However, by decreasing the coverage, i.e., by not serving mobiles that are far from the base station and experience heavy path loss, the remaining mobiles would be able to send with higher power and achieve a higher throughput, hence the aggregate utility of the remaining mobiles would increase.

In this section we present numerical investigations that explore the above tradeoff between coverage and social welfare, and how it is affected by the various parameters of the system. Note that we consider a single cell, and do not take into account the interference caused by neighboring cells.

We first describe the propagation model, which determines the path gains<sup>15</sup>, and then the distribution of mobile users as a function of the distance from the base station.

The propagation model considered is the Okumura-Hata model [12], which for an urban macrocell becomes

$$L = 137.4 + 35.2 \log_{10}(d), \quad (46)$$

where  $L$  is the path loss in dB and  $d$  the distance from the base station in Km. For a suburban microcell, the model is

$$L = 129.4 + 35.2 \log_{10}(d). \quad (47)$$

Hence, the path gain has the form

$$g(d) = kd^{-u},$$

with  $k_{sub} = 1.1482 \cdot 10^{-13}$  for the suburban environment, and  $k_{urb} = 1.8197 \cdot 10^{-14}$  for the urban environment; the distance exponent is  $u = 3.52$  in both cases.

The cumulative distribution function giving the number of mobiles within a radius of  $d$  (in Km) from the base station is taken to be

$$N(d) = \rho_N \pi d^v.$$

---

<sup>15</sup>In particular, the propagation model captures the path attenuation only. Other path imperfections include shadowing (slow fading) and multipath fading (also known as Rayleigh fading); the latter occurs at fast timescales and is tackled with fast closed-loop power control.

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Table 3: Parameters for social welfare-coverage investigation.  $d$  is distance in Km.

parameter	function	values
noise, $\eta$	-	$10^{-13}$ Watt
max mobile power, $\bar{p}$	-	0.2 (default), 0.6 Watt
path gain, $g(d)$	$kd^{-u}$	$u = 3.52$ , $k_{urb} = 1.8197 \cdot 10^{-14}$ (default), $k_{sub} = 1.1482 \cdot 10^{-13}$
number of mobile users, $N(d)$	$N(d) = \rho_N \pi d^v$	$\rho_N = 10, 20$ (default), 30, $v = 1$ (default), 2
$BER(\gamma)$ (DPSK)	$0.5e^{-\gamma}$	
number of bits per pkt, $L$		60
$\gamma^*$ , from (20) and Fig. 4		5
Utility, $U(x)$	$1 - e^{-bx}$	$b = 0.1$ (default), 0.2

In the experiments we consider the values  $\rho_N = 10, 20, 30$  and  $v = 1, 2$ ; the latter value corresponds to the case where the density of mobile users is constant, independent of the distance from the base station.

The other parameters and values we consider are shown in Table 3.

The numerical investigations are shown in Figures 15-19. In each experiment, we assume that all mobile users have the same utility and maximum power constraint.

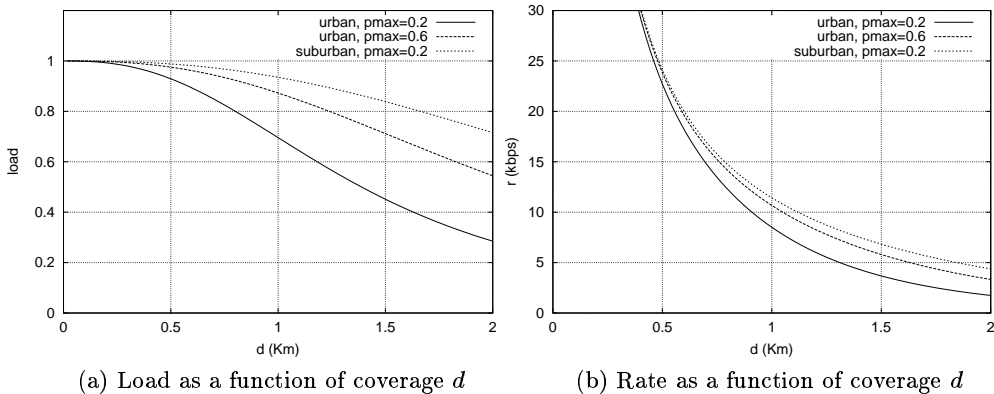


Figure 15: The load and rate are higher in a suburban environment, compared to an urban environment, and for a higher power limit at each mobile.

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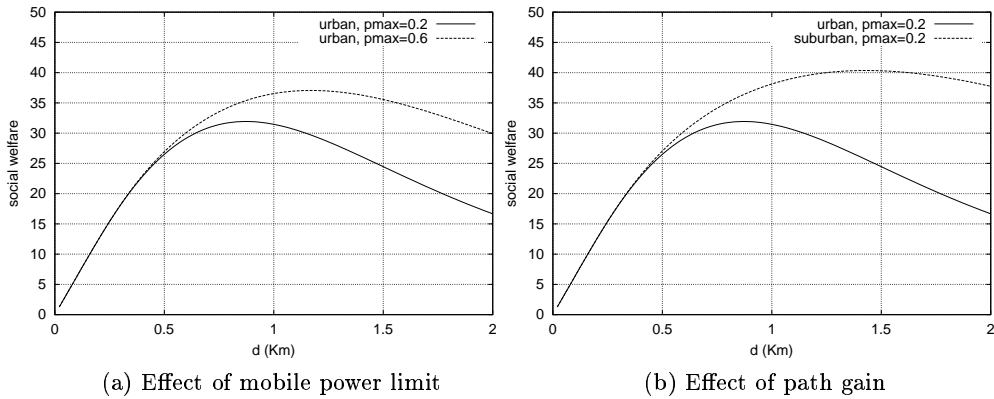


Figure 16: Social welfare as a function of coverage. Observe that the social welfare, as well as the optimal coverage at which the maximum social welfare is achieved, is higher for a higher power limit, and for a higher path gain, which is the case in a suburban compared to an urban environment.

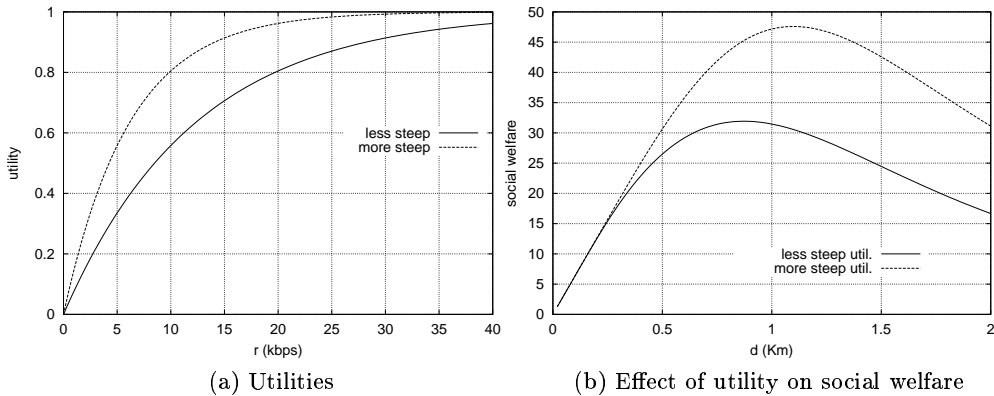


Figure 17: Observe that the social welfare and the optimal coverage at which the maximum social welfare is achieved, is higher for a steeper utility.

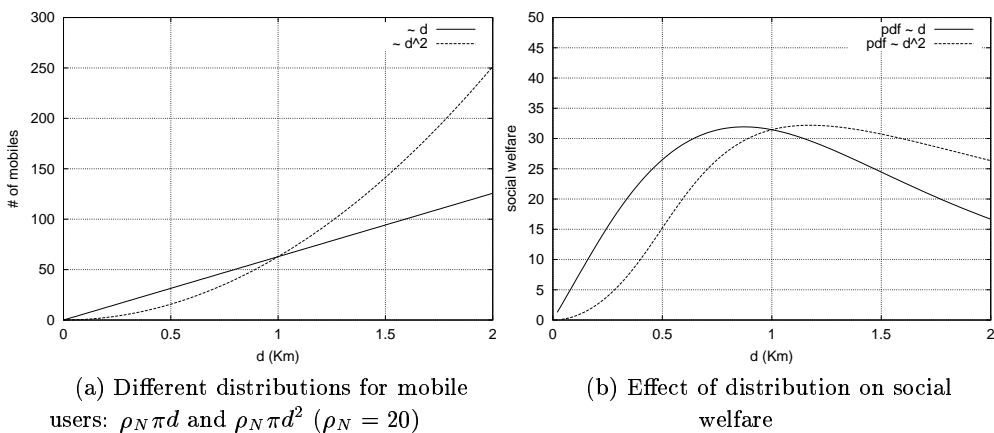


Figure 18: In the case of an equal density distribution, the maximum social welfare is achieved for a larger coverage, compared to the case of a distribution where the density decreases with the distance from the base station. On the other hand, the effect on the value of the social welfare is small.

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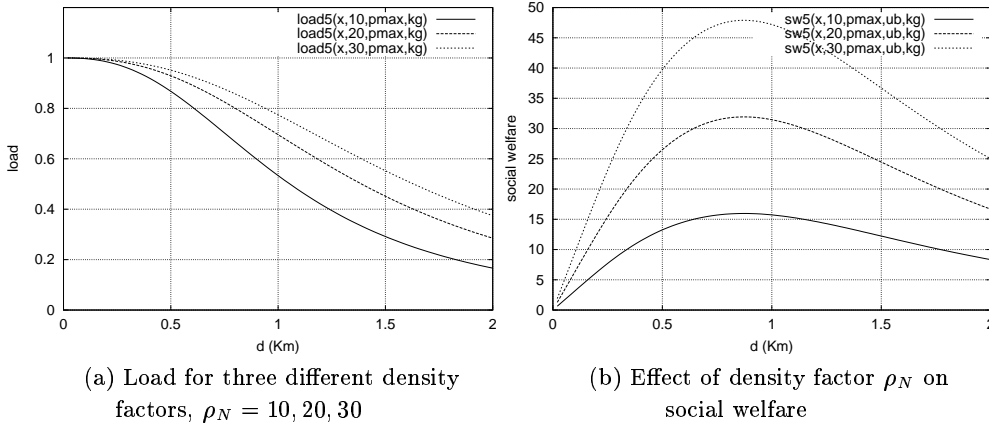


Figure 19: The social welfare is higher for a higher density factor  $\rho_N$ . On the other hand, the coverage for which the social welfare is maximized is not affected.

### B.4 Comparison of economic models for rate-inelastic/quality-elastic traffic

In this subsection we present and discuss numerical investigations comparing the steady state behavior of the approaches in [7, 41] and [45], with the one discussed in Section 5 and given by (34). To simplify things we assume that there are a large number of mobiles. The three different approaches we compare are the following:

- NPGP<sup>16</sup> (Non-cooperative power control game with pricing) [7]. Each mobile user adjusts the target bit-energy-to-interference-density ratio  $\gamma$  to achieve the following maximization:

$$\max_{\gamma} \frac{rP_s(\gamma)}{p} - \lambda_{npgp} p, \quad (48)$$

where  $P_s(\gamma)$  is the packet success rate.<sup>17</sup> We assume that the price per unit of power  $\lambda_{npgp}$  is independent of the load in the wireless network.<sup>18</sup>

- UBPC (Utility-based power control) [45]. The utility-based power control approach in [45] is applied in the downlink; we consider its application to the uplink. Hence, each mobile adjusts the target bit-energy-to-interference-density ratio  $\gamma$ , equivalently its transmission power  $p$ , to achieve the following

<sup>16</sup>[41] considers two versions of the algorithm: In the first, there is no constraint on the minimum  $\gamma$ , hence on the minimum transmission power at the mobile, and in the second there is a minimum. The existence of an equilibrium is proved only for the latter. In practise, it will not be practical to constraint all users to send with a minimum power (they can always turn their mobiles off!); moreover, the minimum  $\gamma_{min}$  can be unrealistically high: Indeed, according to [41], the minimum  $\gamma_{min}$  must satisfy  $\frac{\partial^2 f(\gamma)}{\partial \gamma^2} = 0$ , where  $f(\gamma) = P_s(\gamma)$  which in our case is given by  $BER(\gamma) = 0.5e^{-\gamma}$ ; hence, we have  $\gamma_{min} = \ln L = \ln 60 = 4.09 (= 6.12 \text{ dB})$ . In our investigations, we will consider that such a minimum does not exist; if, however, we were to include the minimum, then the steep decrease of  $\gamma$  to zero in 20 would not exist, and  $\gamma$  would be lower bounded by 4.09.

<sup>17</sup>In [7, 41], a function slightly different from the packet success rate is used; this is done to avoid the degenerate case where the first term in (48) becomes infinite, since  $P_s(0) > 0$ , i.e., the percentage of successful bits is greater than zero, even when  $\gamma$ , hence the power  $p$ , are zero. This will not affect our investigations, so we will simply consider the packet success rate.

<sup>18</sup>The same authors in [3] indicate that the price  $\lambda_{npgp}$  can be taken to be a (linear) function of the transmission rate. In the investigations of this section we consider traffic with a fixed rate, hence such a dependence will not concern us. Moreover, [41] discusses a procedure for obtaining a 'good' value for the price, where 'good' is taken to mean a price that increases the utility for every user, hence the Pareto efficiency of the whole system.

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maximization:

$$\max_{\gamma_i} U_i(\gamma_i) - \lambda_{ubpc}^i p_i. \quad (49)$$

The price per unit of power  $\lambda_{ubpc}^i$  can be taken to reflect the congestion experienced by a user, in which case the following formula is proposed

$$\lambda_{ubpc}^i = a_{ubpc}(I_i + \eta), \quad (50)$$

where  $a_{ubpc}$  is a constant and  $I_i$  is the interference experienced by mobile  $i$  due to the transmissions to the other mobiles. From the above, note that the price per unit of bandwidth can be different for different mobile users.

In our investigations we consider the case of a large number of mobile users, hence  $I_i \approx I$ , for all  $i$ , where  $I$  is the sum of the power of all signals received at the base station. If  $\rho = \sum_i \alpha_i$  is the total load, then  $I + \eta = I_{total}$  combined with (6) give

$$I_i + \eta \approx I_{total} = \frac{\eta}{1 - \rho}.$$

Equation (49) is identical to (27), which holds for the downlink, for which there is a bound on the maximum transmission power at the base station. Indeed, to reflect congestion in the downlink, prices should be an increasing function of the load, determined by the sum of powers  $\sum p_i$ , rather than (50), where prices depend on the interference a mobile experiences.

- CP (Congestion pricing). In this scheme the user problem is given by (34):

$$\max_{\gamma_i} U_i(\gamma_i) - \lambda r_i \gamma_i. \quad (51)$$

Moreover, we assume that the price per unit of resource  $\lambda$  is given by

$$\lambda = \frac{a_{cp}}{1 - \rho}, \quad (52)$$

where as above,  $\rho = \sum_i \alpha_i$  is the total load.

Since the three schemes described above have quite different objectives, rather than comparing them in terms of aggregate measures such as the sum of utilities, we will compare them in terms of the target  $E_b/N_0$ , utility, power, and charge in the steady state, and how these depend on the distance of the mobile from the base station and the total load in the wireless network.

For the numerical comparisons, the price  $a_{cp}$  (= 1.225 per Kbps) was selected so that in the equilibrium the load is  $\rho = 0.60$ , when there are  $N = 40$  mobiles, all with the same utility  $U(\gamma) = 1 - e^{-0.8\gamma}$  and rate  $r = 10$  Kbps; moreover, the price  $a_{ubpc}$  (= 1023 per mW) was selected so that at distance  $d_0 = 0.5$  Km, the UBPC gives the same target  $\gamma$  as the CP scheme. Finally, we set  $\lambda_{npgp} = \frac{a_{ubpc}\eta}{1-\rho} = 2.558$  (per Watt). Hence, the price per unit of power in both the NPGP and the UBPC scheme, for a given total load, is the same; note, however, that the price in the UBPC scheme depends on the load, whereas in the NPGP scheme it is independent.

Finally, the path model considered is given by (46), which corresponds to an urban environment.

### Dependence of $\gamma$ on the distance

Figure 20 shows that for CP, the target bit-energy-to-noise-density ratio  $\gamma$  is independent of the mobile's distance from the base station, since the charge does not depend on the distance. On the other hand, for UBPC, charges depend on the transmitted power; as a result,  $\gamma$  decreases with the distance. Finally, for NPGP,

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$\gamma$  decreases slightly with the distance, and jumps to zero at some point; at the distance where this jump occurs, sending even with a small power results in a negative net utility, hence it is preferable not to send at all.

The figure also shows that, for UBPC, with a less steep utility ( $1 - e^{-0.4\gamma}$  is less steep compared to  $1 - e^{-0.8\gamma}$ ), the dependence of  $\gamma$  with the distance is steeper. For CP, a less steeper utility results in a larger value for  $\gamma$ .

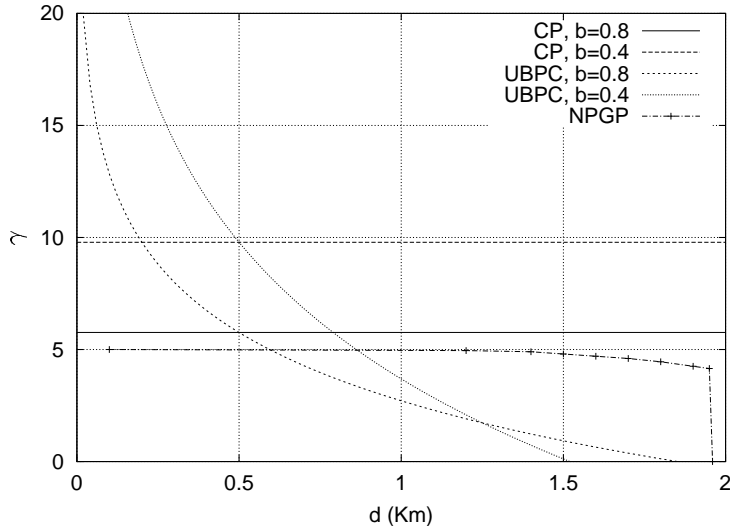


Figure 20: For CP,  $\gamma$  is independent of the distance, since charges are independent of the distance. On the other hand, for UBPC, charges depend on the transmitted power, and  $\gamma$  decreases with the distance. For NPGP,  $\gamma$  decreases only slightly with the distance, and at some point jumps to zero. ( $b$  is factor that appears in the exponent of the utility)

### Dependence of the utility on the distance

Figure 21 shows that the utility of a user in the CP approach is independent of the distance, which is expected since  $\gamma$  is also independent of the distance, Figure 20. On the other hand, for UBPC, the utility is a decreasing and concave function of the distance. Finally, for NPGP the utility is a decreasing and convex function of the distance.

### Dependence of the power on the distance

Figure 22 shows that under the CP approach, a mobile's transmission power increases fast with the distance; this is because to achieve a constant  $\gamma$ , the power must increase with the distance to balance the increased path loss. With the UBPC approach, the power initially increases with the distance. This is due to the initial convex dependence of  $\gamma$  on the distance. For example, assume  $\gamma \propto r^{-1}$ . From (2) we have (assuming a large number of mobiles)

$$p \approx (I + \eta) \frac{r}{W} \frac{\gamma}{g}.$$

since  $g \propto d^{-3.52}$ , see (46), from the last equation we have that  $p \propto d^{2.52}$ ; hence, for small distances, the power increases with the distance.

After some distance, the dependence of  $\gamma$  on the distance becomes approximately linear, and as a result the power decreases with the distance. For example, if

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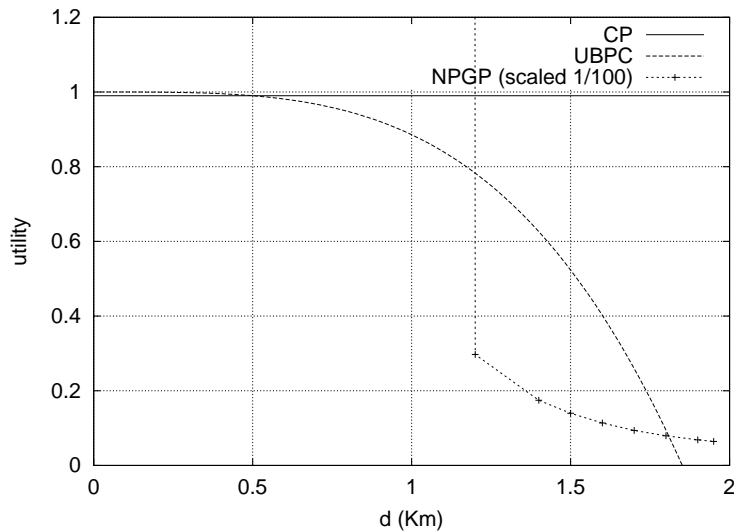


Figure 21: For CP, the utility is independent of the distance, which is expected since  $\gamma$  is also independent of the distance. On the other hand, for UBPC, the utility is a concave function of the distance, whereas for NPGP the utility is a convex function of the distance.

$\gamma \propto 1 - ad$  (where  $a > 0$ ), then using the last equation we have that  $p \propto d^{3.52} - ad^{4.52}$ ; hence, after some distance, the power decreases with the distance.

For NPGP, the power initially increases very fast with the distance. The behavior of the power is similar in the CP scheme, only that the rate of increase in the CP scheme is slightly higher than in the NPGP scheme; this is because in the NPGP scheme  $\gamma$  decreases slightly with the distance. At some distance, for NPGP,  $\gamma$  falls to zero, hence so does the power.

### Dependence of the charge on the distance

For CP, the charge is independent of the distance. For UBPC and NPGP, the dependence of the charge with the distance is similar to the dependence of the power with the distance, Figure 22, since charges are proportional to the power.

Up to now, we have assumed a fixed load. Next we investigate the behavior of the three algorithms when the network load changes.

### Dependence of $\gamma$ on the load

Figure 24 shows that  $\gamma$  depends more on the load for UBPC than for CP. On the other hand,  $\gamma$  for NPGP is independent on the load, since the price is independent of the load.

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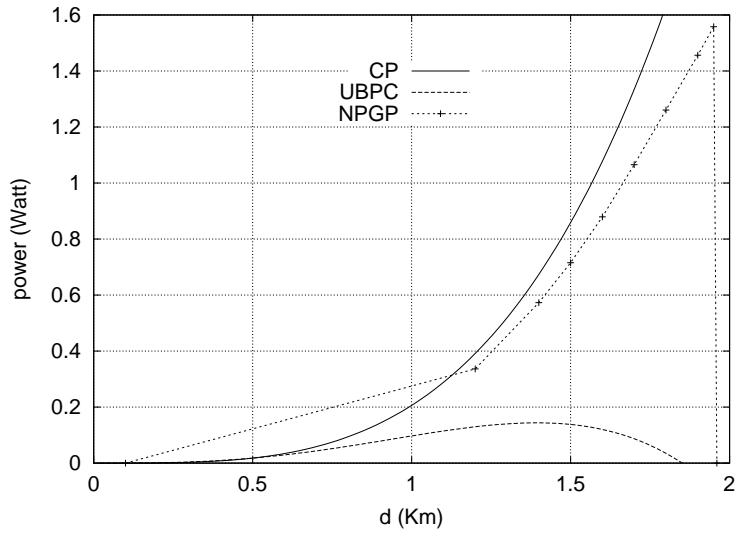


Figure 22: The power for UBPC initially increases, but then starts to decrease; this is due to the combination of two effects:  $\gamma$  decreases with the distance, as shown in Figure 20, but also the path loss increases.

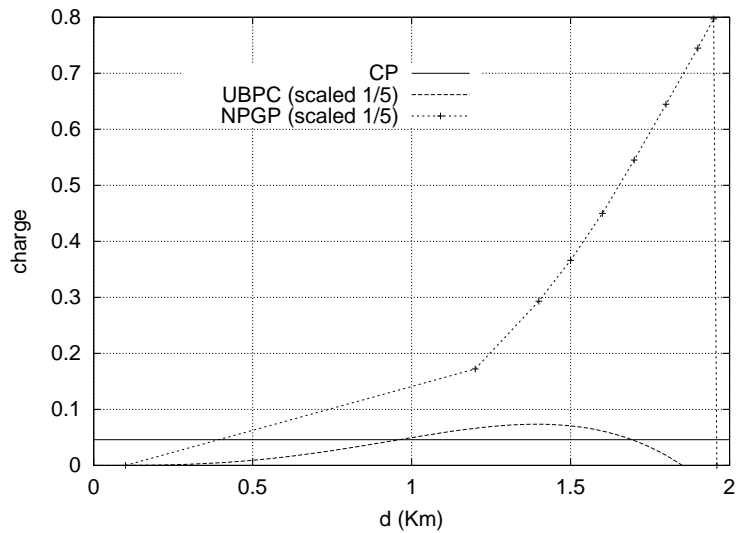


Figure 23: For CP, the charge is independent of the distance. For NPGP and UBPC, the shape of the curves is similar to that of Figure 22, which shows the power as a function of the distance.

*Congestion Pricing for Resource Control in WCDMA*

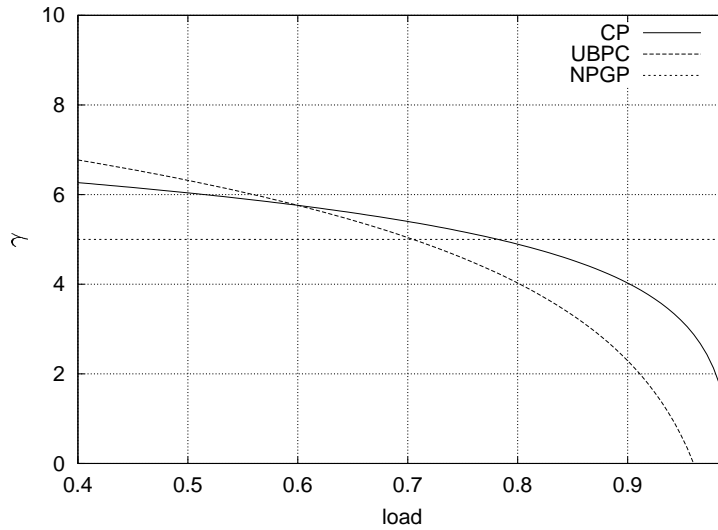


Figure 24: For NPGP,  $\gamma$  is independent of the load, whereas for both UBPC and CP, it decreases with the load. Indeed, for UBPC the dependence is greater, and  $\gamma$  hits zero before the utilization reaches 1.

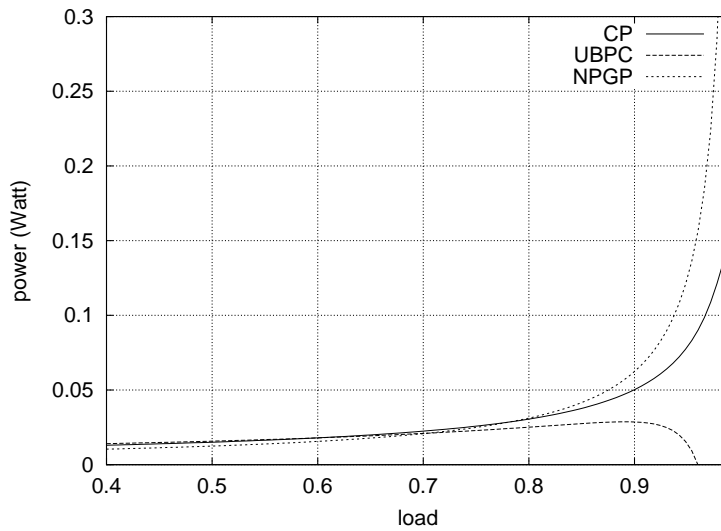


Figure 25: For NPGP and CP, the power increases with the load. For UBPC, it initially increases slightly, but then drops to zero; this is due to the behavior of  $\gamma$  shown in Figure 24.

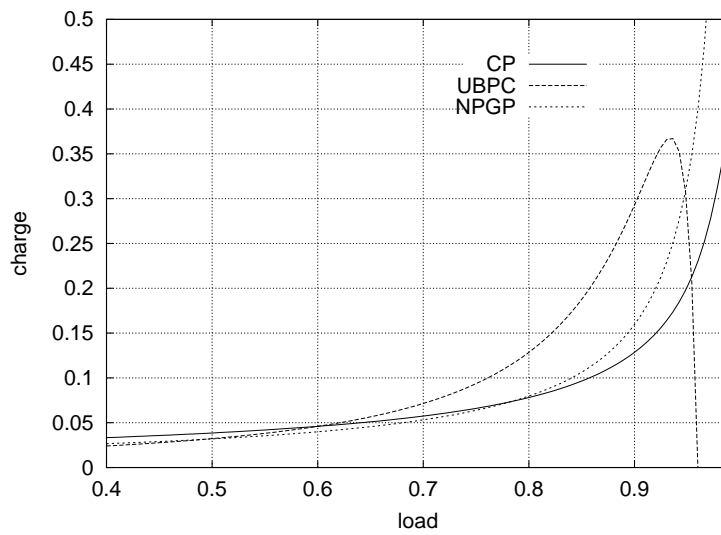


Figure 26: For NPGP and CP, the charge increases with the load. For UBPC, it initially increases, but then drops to zero; this behavior is similar to the behavior of the power, Figure 25, and the behavior of  $\gamma$ , Figure 24.