On Multi-layer Modeling and Analysis of Wireless Access Markets

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Abstract—Advances in networking and regulatory changes on access and competition rules enable new network architectures, service paradigms, and partnerships, opening new opportunities for business cases. Unlike traditional cellular-based markets, new spectrum and wireless access markets are formed that have larger sizes, are more diverse, and can offer an improved set of services. The analysis of such markets is challenging due to a plethora of phenomena that manifest in different spatio-temporal scales. The main objective of this work is the development of a modular multi-layer modeling framework and simulation platform for analyzing wireless access markets. This framework employs game theory and queueing-theoretical models to instantiate a market at multiple spatio-temporal scales. At a microscopic layer, it models each entity of the market in a fine level of detail. By applying various aggregations, it also models the average behavior of certain clusters of entities. In that way, it can analyze a certain phenomenon at the appropriate level of detail, addressing the tradeoff between the loss of accuracy and computational complexity. The analysis then focuses on the flex service, a novel paradigm which allows users to select their provider dynamically. The proposed framework is used to model and analyze the performance of markets that offer the flex service. It employs various metrics, such as blocking probability, percentage of disconnected users, social welfare, and profit, to assess whether this service is beneficial to users, regulators, and providers, respectively. Furthermore, it highlights various challenges in modeling such markets and demonstrates the advantages in using the proposed multi-layer framework.

Index Terms—Wireless networks, network economics, access markets, microscopic & macroscopic modeling, simulation

1 INTRODUCTION

The recent technological innovations and regulatory changes, along with the rapid growth of mobile devices and demand, trigger drastic changes in wireless access markets. New access markets of larger size (in the number of users and providers), more heterogeneous (in the user population and services), and more dynamic are formed. Although traditionally users subscribe to or prepay for specific cellular operators/providers for network access, the technology that allows users to dynamically select their operator is available. Moreover, in recent reports, the Body of European Regulators for Electronic Communications (BEREC) envisages measures for roaming users and consumer empowerment to boost consumer choices.

As wireless access and use increase, consumers are differentiated by their usage, data-rate requirements, and willingness to pay. Inevitably subscribers with relatively high demand are subsidized by the ones with low demand. As the wireless network technology advances, a more diverse set of services is made available. To this end, we introduced the paradigm of a “flex user” who is not “locked” to a specific provider but can dynamically select base stations (BSs) of different providers based on various criteria, such as network conditions, and offered prices. Specifically, flex users are flexible to select the appropriate provider even on a per-session basis. During a session, they transmit and receive data via a BS. This flex service paradigm, which has been assumed as a typical access paradigm in wireless LANs, could be a new type of service offered in cellular markets. A similar concept is the “soft” (or virtual) SIM cards. The flex service could be also viewed in the context of a user with multiple SIM cards available.

To assess the performance of the flex service in a business-driven manner, we have developed a game-theoretical modeling framework and simulation platform. We model the market with two distinct games, the first modeling the competition of providers, through the price setting, and the second the user service selection. Providers aim to optimize their revenue by determining the prices of their services, while users dynamically decide to buy a long-term subscription or become flex users. The user selection of a certain service, namely, a long-term subscription or a flex service, is valid for a certain time period, after which, the user renews its decision. Each user is modeled as a distinct entity with its demand, preferences, and

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utility. The analysis has demonstrated the benefits of the flex service, in terms of blocking probability and percentage of disconnected users. Moreover, the computational and scalability issues when analysing such markets for long time periods or at a nation-wide level are prominent. It becomes important to be able to select the appropriate level of detail for modeling a market of interest, depending on the requirements of the specific study, quantifying the computational complexity and accuracy tradeoff. This need motivated further our research. The paper develops a modular multi-layer modeling framework, including models at multiple levels of detail: a microscopic model describes each distinct entity (e.g., user), while mesoscopic or macroscopic models define aggregations of entities (e.g., homogeneous user populations).

To the best of our knowledge, it is the first paper that develops a multi-layer modeling of access markets, quantifies these tradeoffs between accuracy and computational complexity, and provides a methodology for selecting the appropriate level.

Our earlier work [1], [2] modeled and analyzed a wireless access market at a microscopic level. This paper extends the modeling framework to incorporate multiple levels of detail. It designs the mesoscopic and macroscopic levels by applying aggregations and clustering algorithms. Using this extended framework, it models a market with subscriptions and the flex service. It also employs a more expressive and amenable to theoretical analysis set of metrics, utility functions, and customer types than our earlier work. It comparatively analyzes markets with the flex service and subscriptions and validates the earlier results and main trends. The analysis demonstrates the benefits of the flex service paradigm, using network economics and offers insights to regulators, users, and providers. Finally, it evaluates the performance of the framework, discussing its computational complexity and quantifying the tradeoffs between accuracy and computational complexity at various levels.

The paper is structured as follows: Section 2 overviews the related work. Section 3 describes the main components of a wireless access market, while Section 4 presents the multi-layer modeling framework and the simulation platform that instantiates it. Section 5 focuses on the performance analysis, presenting in Section 5.1 the simulation testbed and in Sections 5.2, 5.3, and 5.4 the performance of microscopic, macroscopic, and mesoscopic levels, respectively, and the tradeoffs between accuracy and computational complexity. Finally, Section 6 summarizes our conclusions and future work plan.

2 RELATED WORK

Existing models of wireless access markets can be classified into two general categories, the microscopic- and macroscopic-level ones. Microscopic-level approaches model each entity, and its interactions with other participating entities, at a fine level of detail. However, due to the high computational complexity, they typically assume a limited number of such entities. For example, some studies [3]–[12], model each user as a distinct entity with its own profile which depends on its willingness to pay or target quality of service (QoS). On the other hand, macroscopic-level approaches model the “average” behavior of certain types of entities (e.g., user population, network infrastructure) to make the analysis more tractable [13]–[17]. In contrast to these approaches, our paper develops a complete multi-layer framework that allows the instantiation of a wireless access market at multiple levels of detail.

In general, studies on wireless markets often focus on either the spectrum acquisition or the service provisioning. Spectrum acquisition involves the process with which providers acquire licenses from the state or other providers to operate at certain portions of the spectrum. Service provisioning focuses on the support of various services in the market, the price determination, and the end-user decision making. The related work on spectrum acquisition makes various assumptions. For example, some of the studies [18], [19] focus on nation-wide licenses that are valid for a long period of time, while others describe various spectrum-leasing mechanisms, in which license holders may sublease a portion of their spectrum to other firms, at specific regions and for shorter periods of time [13], [16], [20], [21].

Another recent trend in wireless access markets is the presence of mobile virtual network operators (MVNOs). MVNOs do not own spectrum or network infrastructure but sublease network resources from other providers [22], [23]. They typically target specific user populations or regions. The femtocell paradigm also has received considerable attention. For example, Yi et al. [24] proposed a market scenario in which a femtocell provider subleases spectrum from a macrocell provider. Kang et al. [25] analyzed a macrocell provider that allows multiple femtocell users to function within its frequency band and charges them based on the interference power.

Some service provisioning studies consider a provider that offers two types of services, one for primary and one for secondary users. Wysocki et al. [12] assumed that the price for the secondary service is determined in a way that balances out the QoS degradation of the primary service, while Chang et al. [9] applied a reservation scheme to guarantee the quality of the primary service. Other studies [3], [7] analyzed the service provisioning in CDMA wireless networks, in which users are charged according to their transmission power, while Gao et al. [4] considered a provider that offers services of different quality and price. In [17], multiple providers offer services to multiple groups of users. Yang et al. [10]
study a market in which multiple primary users offer spectrum opportunities to a set of secondary users that compete for wireless access based on the slotted Aloha or CSMA protocol. Note that most of the above referenced papers focus on different types of markets (e.g., spectrum markets with primary and secondary users) compared to the one considered in this work. The models presented in these papers are either purely microscopic or macroscopic. Although, this work focuses on the flex service, the proposed multi-layer modeling methodology could also be applied to other types of markets (e.g., spectrum markets).

3 SYSTEMS MODEL AND ASSUMPTIONS

Wireless access markets are complex systems that involve various economic and technological parameters. In such markets, there are mainly two types of entities, the providers and users. A provider has deployed a cellular infrastructure and offers subscription and flex services to users. Two important decision making processes take place: (A) Periodically, providers set the prices of their services aiming to maximize their revenue. (B) Each user periodically selects the most appropriate provider and service. While a user has an active subscription or flex service, it initiates sessions. During a session, a user transmits and receives data via a BS. The price for a service as well as the service selection of a user last for the time period of an epoch. Note that in our framework, the epoch has a fixed duration and lasts for several days (or months). A session lasts for several minutes (Fig. 3).

For an educated selection of the service and provider, users should consider their traffic demand, willingness to pay, and requirements, as well as the cost of the service. However, a typical user is not willing to process such complex information. A software agent running on the user mobile device could in a (semi-)automatic manner select the best service on behalf of the user. In this work, we assume the presence of such a software agent, called client throughout the paper. We also assume the existence of a server attached to a data repository that collects measurements about the sessions, traffic demand, and user profiles through a crowdsourcing monitoring system called u-map. The client uploads measurements about its sessions on the u-map server. Statistics on u-map data are available to clients and providers. We have developed and evaluated the u-map prototype [26]. This paper uses u-map to represent the collected knowledge about the users and performance of services. The main components of the market are presented in Fig. 1 and are described in more detail in the following subsections.

Infrastructure: Each provider has deployed a cellular topology that offers wireless access via its BSs to clients in a small city. Providers divide their channels into time slots according to TDMA. The interference power at a BS during a time slot is computed considering the contribution of all interfering devices at co-channel BSs.

Clients: To start a session, a client generates a request to connect to a BS. The duration of a session and the off duration (i.e., the time interval between the end of a session and the start of the immediately next one of the same client) are given by theoretical distributions. To initiate a session, the client needs to first select a BS. The BS selection is based on either the data-rate or price. Specifically, a price-conscious client selects the BS that minimizes its cost spending, while a QoS-conscious client selects the BS that maximizes its achievable data rate. Furthermore, each client is characterized by a data-rate threshold \((R_u)\) which is the minimum required data rate. When a client cannot satisfy its data-rate requirement with any BS in its neighborhood, its session becomes blocked.

Service types: Two service types are considered, the subscription and flex service. Periodically, a client is required to select a service type or remain disconnected. During a disconnection period, the client does not have any active service, and thus, cannot initiate sessions. A subscriber or flex user needs to first select a BS in order to start a session. A flex user may select a BS of any provider, while a subscriber of a certain provider may select only BSs of that provider. We assume that a flex user has a “basic” subscription with the network infrastructure of each provider in order to receive notifications, incoming calls or other data sessions. However, in order to initiate and complete a session, a flex user is required to select a provider dynamically.

Charging: The subscription and flex services are charged according to different schemes: The pricing for subscriptions is a flat rate scheme that charges the user with a specific fixed cost (per epoch). On the other hand, the flex users are charged for each session in a cost-per-minute manner. The service (i.e., flex service, or subscription) or the disconnection lasts for an epoch. The selection of the service type is performed once during each epoch. The BS selection
takes place before the start of each session. Fig. 2 illustrates the decision-making mechanism of a typical subscriber and a typical flex user in a wireless access duopoly.

**u-map:** u-map is modeled as a data structure which stores information about users and providers. Periodically, each client reports the status (successful or blocked) and duration of its sessions along with its service type, client id, provider id, and constraints. Clients obtain information about the average blocking probability per service, which is computed as the percentage of the blocked sessions that were made during the last \( w \) epochs, as reported by clients of that service in u-map.

### 4 Game-theoretical Modeling

The evolution and performance of wireless access markets involve networking and business aspects that manifest various spatio-temporal dependencies and localities. For example, user profiles, network topology, and channel conditions may vary across regions. Also, local decisions of users and providers may be required. This motivated us to develop a game-theoretical modeling framework and simulation platform that encompass multiple levels of detail, from a microscopic level to a macroscopic one.

The framework considers a wireless access market that consists of two providers \( P = \{1, 2\} \) and a population of \( n \) users \( U = \{1, ... , n\} \). This market is modeled using two distinct games, one for the competition of providers and one for the user decision making. The game of providers takes place at epochs (i.e., time periods of fixed duration). At the beginning of each epoch, providers adapt the prices for their services aiming to maximize their revenue. Each user selects a type of service depending on its price and performance (according to a utility function).

Our modeling framework is configurable and parameterized based on the channel, infrastructure and network topology, type of users (e.g., service, demand, mobility, constraints, preferences), providers (e.g., price adaptation, services), and available information. The simulation environment that implements this framework is also modular, in that, it can instantiate and implement different models for the aforementioned parameters. The games of users and providers are described in detail in the following subsections.

#### 4.1 Game of Users

The service selection of users is modeled as a population game. Each member of the population can choose among four available strategies \( H = \{1, 2, 3, 4\} \). Strategies 1 and 2 correspond to subscriptions with Providers 1 and 2, respectively, strategy 3 indicates the flex service, and strategy 4 denotes the disconnection state. All users have the same options with respect to service selection, and as such, the set of all possible combinations of user choices (i.e., set of user strategy profiles) is defined as \( S = H^n \). The selection of a strategy (i.e., service) is based on a utility function that expresses the tradeoff between quality of service and price. In this work, the utility function is a linear combination of blocking probability and price.

\[
\pi_{ui}(s) = \begin{cases} 
-B_i(s^u_i) - b_u c^s_{i} & \text{if } i = 1, 2 \\
-B_i(s^w_i) - b_u c^s_{i}(s^w_i) & \text{if } i = 3 \\
-k_u & \text{if } i = 4 
\end{cases}, \quad (1)
\]

**Utility function:** Eq. 1 defines the utility of a user \( u \in U \), when the strategy profile of all users is \( s \in S \) and the user \( u \) switches to strategy \( i \in H \) from its current service. The strategy profile resulting from this transition is defined as \( s^u_i \). \( B_i(s^u_i) \) is the blocking probability that is associated with the service \( i \), and \( c^s_i \) is the subscription rate of Provider \( i \). The client estimates the price that the user \( u \) pays per epoch when it selects the flex service \( c^u_i(s^w_i) = (a^u_i c^s_{i1} + a^u_i c^s_{i2})d_u (1 - B_3(s^w_i)) \), where \( d_u \) is the average total duration of all sessions of user \( u \) per epoch. The probabilities \( a^u_i \) and \( a^w_i \) indicate the frequencies with which user \( u \) has selected Providers 1 and 2, respectively, during the last \( w \) epochs (as reported by u-map). The duration \( d_u \) is divided between providers according to the probabilities \( a^u_i \) and \( a^w_i \). The total cost is estimated based on the time spent at the network of each provider and their flex rates \( (c^s_{i1} \) and \( c^s_{i2} \), respectively). Only non-blocked sessions contribute to the total payment (indicated by the term \( (1 - B_3(s^w_i)) \)). All the parameters of Eq. 1 are estimated with the data collected in u-map.

The user profile is expressed by the utility function, which is parameterized based on the price sensitivity \( (b_u) \), disconnection threshold \( (k_u) \), and average total session duration per epoch \( (d_u) \) of a user \( (u) \), respectively. The price sensitivity is a positive value that indicates the importance of price compared to blocking probability in the user utility function. The higher the price sensitivity, the larger the significance of price on the user decision making. The disconnection threshold
is the minimum utility acceptable by a user. If this utility can not be achieved with any of the offered services, the client remains disconnected for an epoch. Finally, the average total session duration per epoch is the average demand in minutes that a user generates per epoch. The profile of a user \( u \in U \) with respect to service selection is annotated as \((b_u, -k_u, d_u)\). In summary, the game of users can now be defined by the triplet \( g = (U, S, \{\tau_{ui}\}_{u \in U, i \in H}) \).

The analytical study of the user population game is challenging. For the estimation of the Nash equilibriums (NEs), a closed form of the blocking probabilities is required. However the analytical estimation of the blocking probabilities in the presence of the flex service is difficult even when all users are identical with respect to their profile (see Section 4.3). For that reason, we proceed to study the game of users via simulations. Specifically, for a given set of offered prices, we simulate the user movement, session generation, and service selection. During service selection, a client first estimates the total cost for each service as well as the corresponding blocking probabilities. For that, we assume that the client can approximately estimate the average total duration of its sessions per epoch based on the history of performed sessions via u-map. Then, it checks whether the utilities of the various services are higher than its disconnection threshold \((-k_u)\). The blocking probability of subscribers of a certain provider is estimated as the percentage of blocked sessions performed by all subscribers of that provider during the last \( w \) epochs at u-map. The blocking probability for the flex service is estimated as the percentage of blocked sessions performed by all flex users during the last \( w \) epochs at u-map. The client selects the service with the highest utility value above the disconnection threshold. If there is no such service, the user becomes disconnected for an epoch.

After the service-type selection, a user who is not disconnected initiates sessions. For each session, a client first selects a BS based on either data rate or price. As mentioned earlier, price-conscious clients select the BS that minimizes their cost spending, while data rate conscious clients select the BS that maximizes their achievable data rate. The decision making of users is shown in Fig. 3.

### 4.2 Game of providers

The competition of providers is modeled as a normal-form game \((P, C, \{\sigma_P\}_{P \in P})\), where \( P \) is the set of providers and \( C \) is the set of strategy profiles. A strategy profile \( c \in C \) contains the offered subscription and flex rates of both providers \( c = (c_1, c_2, c_1', c_2') \), where \( c_1 \) and \( c_2 \) are the subscription and flex rates of provider \( p \), respectively. Each provider can choose its subscription and flex rates from two finite and discrete sets of prices \( C^s \) and \( C^f \), respectively and as such the game is finite. The utility function of each provider

\[
p \in P \text{ is defined as } \sigma_p(c) = U_p^s(c) + U_p^f(c), \text{ where } U_p^s(c) \text{ is the total revenue from subscribers and } U_p^f(c) \text{ is the total revenue from flex users.}
\]

**Empirical game:** The lack of a closed form solution for the equilibrium of the user population game prevents us from estimating closed form solutions for the functions \( U_p^s(c) \) and \( U_p^f(c) \). For that reason, we define the game of providers as an empirical game [27]. An empirical game is exactly the same as a normal-form game with the difference that there is no analytical expression for the utility functions of players. Instead, it provides a game simulator \( \Theta \) that for any strategy profile \( c \in C \), it reports the corresponding revenues \( \{\sigma_p(c)\}_{p \in P} \). A formal definition of the empirical game is \( G = (P, C, \Theta) \). Various algorithms have been proposed in the literature to solve empirical games (estimate pure strategy NEs). Examples of such algorithms are (tabu) best response [28], and minimum regret first search [29]. In this paper, we use the best response due to its simplicity, although other algorithms can be incorporated easily. Specifically, at the beginning of each epoch providers adapt their rates in two phases: their subscription rate at the first phase and their flex rate at the second phase. At the first phase, all providers (in parallel) simulate the market for all possible subscription rates they can offer keeping the subscription rate of the other provider and flex rates unchanged. Each provider selects the subscription rate that maximizes its immediate revenue. In the second phase, a similar process is followed: all providers (in parallel) simulate the market, computing their profit for each flex rate they can offer, keeping fixed the flex rate of the other provider and subscription rates. Then, each provider selects the flex rate that maximizes its immediate revenue.

In contrast to classical normal-form games, empirical games are analyzed via simulations. Therefore, the computational complexity of the simulator \( \Theta \) is an important parameter. In our case, at the microscopic
level, the game simulator generates a large number of events which increases the execution time for the best response algorithm in the order of several days! This motivated us to explore the multi-layer aspect: model the game of users (service selection) at a mesoscopic or macroscopic level to reduce the computational complexity of the game simulator.

### 4.3 Multi-layer modeling

As mentioned earlier, the microscopic modeling framework describes each user as a distinct entity. However, the diversity and large size of the user population makes the analysis challenging due to the estimation of many different utility functions. To reduce the computational complexity of the game simulator, various aggregations are performed. Specifically, we divide the user population into a set of clusters $U_j = \{1, ..., J\}$, each corresponding to a “homogeneous” population, i.e., its members are characterized by the same representative profile. The representative profile of a cluster $j \in U_j$, denoted as $(b_j, -k_j, d_j)$, corresponds to the centroid of that cluster, as reported by a clustering algorithm on the profiles of all users of the microscopic modeling. As mentioned, the profile of each user $u$ is modeled as a vector $(b_u, -k_u, d_u)$ containing the price sensitivity, disconnection threshold, and average total session duration per epoch of that user. This paper employs the K-means algorithm [30]. The aim of this algorithm is to partition the user profiles into $(J)$ clusters such that each user profile belongs to the cluster with the nearest centroid. The number of user clusters $(J)$ determines the mesoscopic level. Fig. 4 illustrates the transformation of the user population from the microscopic level to the mesoscopic level via clustering. Tables 1 and 2 summarize the main parameters of the modeling framework at these levels.

Due to the homogeneity of a cluster of users, all its members are “assigned” the same strategy, when selecting the same strategy. Unlike the microscopic level that simulates the decisions of each user with respect to service, the mesoscopic levels estimate the percentage of users of each cluster that choose each service. That is, for each cluster, the strategies of its members can be represented by a probability distribution over the set of strategies $(H)$. The set of all probability distributions over $H$ is defined as $\Delta(H)$, and the set of strategy profiles of all clusters is $S_J = \Delta(H)^J$. The utility of the users of cluster $j$ when they select the strategy $i \in H$ is defined as follows:

$$
\pi_{ji}(z) = \begin{cases} 
-B_i(z) - b_j c_i^s & \text{if } i = 1, 2 \\
-B_i(z) - b_j c_i^f(z) & \text{if } i = 3, 4 \\
-k_j & \text{if } i = 4
\end{cases}
$$

In Eq. 2, $z \in S_J$ is the user strategy profile. It is composed by $J$ probability distributions $(z = (z_1, ..., z_J))$, one for each cluster. The probability distribution $z_j = (z_{j1}, ..., z_{j4})$ shows the distribution of users of cluster $j$ across the available strategies $i \in H$. Indices 1 and 2 correspond to subscriptions with Providers 1 and 2, respectively, index 3 indicates the flex service, and index 4 denotes the disconnection state. $B_i(z)$ is the blocking probability associated with the service $i$ and $c_i^f(z)$ is the price that a flex user in cluster $j$ pays per epoch $c_i^f(z) = (a_i^f d_j^f + a_i^s d_j^s) d_j^f (1 - B_3(z))$. The total session duration of any flex user of cluster $j$ per epoch $(d_j^f)$ corresponds to the total session duration spent at the network of each provider. The probabilities $a_i^f$ and $a_i^s$ indicate the frequencies with which the user

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**TABLE 1: Parameters of the game of users at the micro (meso) level**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_j$</td>
<td>Set of users (clusters) at micro (meso) level</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of users at micro level</td>
</tr>
<tr>
<td>$J$</td>
<td>Number of clusters at meso level</td>
</tr>
<tr>
<td>$H$</td>
<td>Set of user strategies</td>
</tr>
<tr>
<td>$\pi_u(\pi_{ji})$</td>
<td>Utility function of user $u$ (cluster $j$) when selecting strategy $i$</td>
</tr>
<tr>
<td>$B_i$</td>
<td>Blocking probability of service $i$</td>
</tr>
<tr>
<td>$b_j$</td>
<td>Price sensitivity of user $u$ (cluster $j$)</td>
</tr>
<tr>
<td>$-k_j$</td>
<td>Disconnection threshold of user $u$ (cluster $j$)</td>
</tr>
<tr>
<td>$d_j$</td>
<td>Average total session duration per epoch of user $u$ (cluster $j$)</td>
</tr>
<tr>
<td>$a_i^f(\pi_{ji})$</td>
<td>Probability for flex user $u$ (a flex user in cluster $j$) to select Provider $i$</td>
</tr>
</tbody>
</table>

**TABLE 2: Parameters of the game of providers**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>Set of providers</td>
</tr>
<tr>
<td>$c_i^f$</td>
<td>Subscription rate of Provider $i$</td>
</tr>
<tr>
<td>$c_i^s$</td>
<td>Price rate of Provider $i$</td>
</tr>
<tr>
<td>$U_{iP}$</td>
<td>Utility of Provider $p$ from subscribers</td>
</tr>
<tr>
<td>$U_{iP}$</td>
<td>Utility of Provider $p$ from flex users</td>
</tr>
</tbody>
</table>

Fig. 4: Game of users at multiple levels of detail.
chooses Providers 1 and 2, respectively. Thus, the total cost of the flex service is estimated according to the time spent at the network of each provider and the corresponding flex rates ($c_1'$ and $c_2'$, respectively).

Again blocked sessions are ignored for the estimation of the cost. Formally, at the mesoscopic level, the game of users is defined as $g_J = (U_J, S_J, \{\pi_{ji}\}_{j \in U_J, i \in H})$.

The macroscopic level corresponds to a mesoscopic level with only one user cluster. That is, the user population is assumed homogeneous and the user choices are described by one probability vector over the set of strategies. Formally, at the macroscopic level, the definition of the user population game becomes $g_1 = (U_1, S_1, \{\pi_j\}_{j \in H})$. The set $U_1$ contains one user cluster with the representative profile $(b, -k, d)$ which is computed as the centroid of the profiles of users at the microscopic level, and $S_1 = \Delta(H)$.

**Blocking probability model:** To estimate the blocking probabilities at the mesoscopic and macroscopic levels, we employ a simple Markovian queueing-theoretical model. Specifically, we focus on a small region containing one BS per provider. The session and off durations of each user follow exponential distributions. The parameters of the distributions are selected so that the average session generation and service rates over all users coincide with the ones at the microscopic level. The BS of Provider 1 (2) is modeled as a finite queue with $m_1$ ($m_2$) servers, respectively. A server of a BS corresponds to a time slot (in a TDMA scheme). A session is blocked when there are no available slots to serve the user request.

The total session generation rate of all users is $\lambda$ and is composed by the session generation rates of all user clusters. The session generation rate for a cluster $j \in U_J$ is denoted as $\lambda_j^*$. Eq. 3 estimates the session generation rates for subscribers of Providers 1 and 2, $\lambda_1^*$ and $\lambda_2^*$, respectively, and flex users $\lambda'$ by summing the contributions of each user cluster.

$$\lambda_1^* = \sum_{j=1}^{J} z_{j1} \lambda_j^*; \quad \lambda_2^* = \sum_{j=1}^{J} z_{j2} \lambda_j^*; \quad \lambda' = \sum_{j=1}^{J} z_{j3} \lambda_j^* \quad (3)$$

A new session of a subscriber can only be served by the BS of its own provider, while a new session of a flex user can be served by either provider. In general, a flex user selects a BS according to a probability distribution ($\delta, 1 - \delta$). With respect to the BS selection, we consider two types of user preference, the price preference, and rate preference one. In price preference, a flex user selects the BS of the cheapest provider (i.e., $\delta = 1$ if Provider 1 is the cheapest, otherwise $\delta = 0$), while in rate preference, it selects the BS that maximizes its achievable data rate ($\delta$ is the probability that the achievable data rate in the network of Provider 1 is larger compared to the one of Provider 2).

When the BS of one provider is fully utilized, a flex user can only connect to the BS of the other provider assuming that the criteria for the BS selection are satisfied. In general, when there are available slots at both BSs, the session arrival rate at the BS of Provider 1 is $\lambda_1^* = \lambda_1^* + \lambda'$, while when the BS of Provider 2 is fully utilized, the arrival rate for Provider 1 becomes $\lambda_1^* = \lambda_1^* + \lambda'$. Similarly, the arrival rates $\lambda_2^*$ and $\lambda_2'$ of Provider 2 are defined.

A large user population (compared to the number of time slots) is considered. Thus, the total session generation rate is not affected by the number of users being served [31]. We then model the market using a Markov chain with a two dimensional state space as shown in Fig. 5. To estimate the blocking probabilities for the three types of services, the steady-state distribution of the Markov chain is computed:

$$\Phi = (\varphi_{0,0}, \varphi_{1,0}, ..., \varphi_{m_1,0}, ..., \varphi_{0,m_2}, \varphi_{1,m_2}, ..., \varphi_{m_1,m_2}) \quad (4)$$

where $\varphi_{x,y}$ is the steady-state probability for the state $(x, y)$. The probabilities that the BSs of Providers 1 and 2 are fully utilized ($\Phi_1$ and $\Phi_2$, respectively) and the probability that both BSs are fully utilized ($\Phi_3$) are defined as follows:

$$\Phi_1 = \sum_{y=0}^{m_2} \varphi_{m_1,y}, \quad \Phi_2 = \sum_{x=0}^{m_1} \varphi_{x,m_2}, \quad \Phi_3 = \varphi_{m_1,m_2} \quad (5)$$

**TABLE 3: Parameters of the Markov-chain model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda^*$</td>
<td>Total session generation rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Session service rate</td>
</tr>
<tr>
<td>$\lambda_j^*$</td>
<td>Session generation rate for users of cluster $j$</td>
</tr>
<tr>
<td>$\lambda_1^*$</td>
<td>Session generation rate for subscribers of Provider 1</td>
</tr>
<tr>
<td>$\lambda'$</td>
<td>Session generation rate for flex users</td>
</tr>
<tr>
<td>$m_1$</td>
<td>Time slots available at the BS of Provider 1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Probability for flex user to select the BS of Provider 1 when both BSs are not fully utilized</td>
</tr>
</tbody>
</table>

**Fig. 5:** The Markov-chain model of the cellular network. The state $(x, y)$ corresponds to the total number of time slots $x$ and $y$ that serve sessions at BSs of Provider 1 and 2, respectively.
The Markov chain of Fig. 5 corresponds to a “finite-loss system of two queues with overflow” and there is no simple analytical solution for the forward Kolmogorov equations of such a system [32], [33]. On the other hand, numerical methods can be employed to solve the system of Kolmogorov equations. By taking advantage of the structure of the transition rate matrix of the Markov chain, we determine the steady-state probabilities in a low computational cost [34], [35]. In this work, we applied the method proposed in [34].

As in the microscopic level modeling, a user aims to achieve a target data rate at the start of a session. If this data rate cannot be achieved, the session is blocked. The probability that an arbitrary user can achieve its target data rate in the network of Provider $i$ is denoted as $q_i$. The blocking probabilities ($B_i$) for all services ($i = 1, 2, 3$) can now be defined as follows:

$$B_1 = (1 - q_1) + q_1 \Phi_1, \quad B_2 = (1 - q_2) + q_2 \Phi_2$$
$$B_3 = (1 - q_1)(1 - q_2) + q_1(1 - q_2) \Phi_1 + q_2(1 - q_1) \Phi_2 + q_1q_2 \Phi_3$$  \hfill (6)

To estimate the blocking probability of sessions of subscribers of Provider $i$, we distinguish two cases: (1) a subscriber of Provider $i$ cannot achieve its target data rate at the start of a session, which happens with probability $1 - q_i$. In this case, that session becomes blocked, and (2) with probability $q_i$, a subscriber of Provider $i$ can achieve its target data rate. In that case, the session gets blocked only when the BS of Provider $i$ is fully utilized. This happens with probability $\Phi_i$. Therefore, the overall blocking probability for a subscriber of Provider $i$ is $(1 - q_i) + q_i \Phi_i$.

To estimate the blocking probability of a session of a flex user (given by Eq. 6), we can distinguish the following cases: With probability $(1 - q_1)(1 - q_2)$, the flex user cannot achieve the target rate in the network of either provider and its session is blocked. The user achieves its target rate only in the network of Provider 1 (Provider 2) with probability $q_1(1 - q_2)$ (with probability $q_2(1 - q_1)$), and its session becomes blocked with probability $\Phi_1$ ($\Phi_2$), respectively. Finally, with probability $q_1q_2$, the user can achieve its target rate in either of the two networks of providers. In that case, its session gets blocked when the BSs of both providers are fully utilized, which occurs with probability $\Phi_3$.

**User population dynamics:** Based on the user utility function (Eq. 2) and blocking probability model (Eqs. 4, 5, 6), the user population game can be simulated. Specifically, we simulate the user evolution by applying the smoothed best response dynamics [36], also known as Logit dynamics, a system of ordinary differential equations (7).

$$\frac{dz_{ji}(t)}{dt} = r \ast \left( \frac{1}{1 + \sum_{k \neq i} G_{jki}(z(t))} - z_{ji}(t) \right)$$ \hfill (7)

The parameter $r$ controls the speed of the dynamics, while $G_{jki}(z(t))$ are functions of the difference in utility that a user in cluster $j$ achieves when it selects the strategy $\kappa \in H$ compared to when it selects the strategy $i \in H$ (Eq. 8).

$$G_{jki}(z(t)) = \exp \left( \frac{\pi_{jki}(z(t)) - \pi_{ji}(z(t))}{\epsilon} \right)$$ \hfill (8)

The term $\epsilon$ is a noise parameter: when $\epsilon$ tends to zero, users are completely rational, while when $\epsilon$ increases, a degree of irrationality is introduced in the user decision making.

### 4.3.1 Convergence and complexity issues

The existence of a NE in the game of providers is guaranteed, since according to the theorem of Nash, each finite game has at least one NE [37]. For the estimation of a NE, a closed form of the utility functions of providers is required. However, the utilities of providers (i.e., profit) depend on the equilibrium of the user population game. This equilibrium indicates the portion of users that choose each service, which is a metric necessary for the estimation of the utilities of providers. To derive the equilibrium points of the user population game, a closed form of the blocking probability for all services is necessary. The estimation of the blocking probabilities of the various services is based on the queuing-theoretical model presented above. As mentioned, there is no simple analytical solution for the forward Kolmogorov equations of such a model. The lack of a closed form solution for the blocking probabilities prevents the provision of analytical expressions for the equilibrium points of the user population game and for the NEs of the game of providers.

Our scenarios involve complex games for which it is very difficult to derive the expected utilities of providers, even if complete policies for all providers are given. Empirical game theory is a relatively recent research direction in game theory for analyzing such complex games [27]. Various algorithms have been proposed in the literature to solve empirical games and estimate evolutionary stable NEs [28], [29]. These algorithms converge to a pure strategy NE if one exists. In most of the simulation scenarios presented in this paper, the best response algorithm converges to a pure strategy NE. In cases in which the game does not have pure strategy NEs (all NEs are mixed strategy ones), the best response algorithm converges to a periodic solution (market oscillations).

At the beginning of each epoch, providers adapt their prices based on the best response algorithm. Specifically, they simulate the market at the macroscopic level for all possible subscription and flex rates and select the rates that maximize their revenue. The computational complexity of the process is $O(L \times M)$, where $L$ is the number of subscription and flex rate levels and
\section{5 PERFORMANCE EVALUATION}

\subsection{5.1 Simulation setup}

We implemented the modeling framework in Matlab and instantiated various wireless access markets for its performance evaluation. For the analysis of the flex service, we instantiated a wireless access market of a small city, represented by a rectangle of 11 km x 9 km and performed simulations for different scenarios. The parameters of the user profiles remained fixed throughout a simulation scenario. Each experiment (i.e., simulation run) represents the evolution of the market during a period of 50 epochs, each lasting 5 days.

\textit{Wireless network infrastructure:} Each provider has a cellular network that consists of 49 BSs placed on a triangular grid, with a distance between two neighboring sites of 1.6 km. Moreover, each provider owns bandwidth of 5.6 MHz, that is divided into 28 channels of 0.2 MHz width. These channels are allocated to BSs according to a frequency reuse scheme with spatial reuse factors of 4 and 7, for Provider 1 and Provider 2, respectively. Each channel is further divided into three time slots in a TDMA scheme, resulting in 21 time slots per BS of Provider 1 and 12 slots per BS of Provider 2. A single time slot of a given BS can be offered to \textit{only one client}. A client can use \textit{only one} time slot of a given BS, and can be associated with \textit{only one} BS during a given session. Although, this work assumes a TDMA protocol for wireless access, the framework can employ other medium access protocols, such as CSMA or CDMA, by modifying the user utility function appropriately (e.g., using the achievable data rate as an indicator of the QoS). The channel quality is described based on the \textit{Okumura Hata} path-loss model for small cities considering the contribution of shadowing to the channel gain \cite{38, 39}. The maximum transmission power that a client can use is 2 Watts. The chosen parameters correspond to a \textit{typical microcellular network} of a small city \cite{40, 41} and the channel availability of BSs was determined to match the demand of the user population.

\textit{User population:} There are 28000 users in total, distributed according to a uniform distribution in the simulated region of this small city. The parameters of the user profiles follow Gaussian distributions. The mean values of the disconnection threshold ($-k_{u}$) and data-rate threshold ($R_{u}$) are $-0.2$ and $0.1$ Mbps, respectively, while their standard deviation is $0.01$. On the other hand, the price sensitivity ($b_{u}$) is the same for all users (equal to 0.025).\textsuperscript{1} In each scenario, a user selects the BS with the best data rate.

\textit{Client demand:} A client generates a sequence of session requests. The session duration follows a Pareto distribution of mean 5 min (the scale and shape parameters are equal to 3.890 and 4.500, respectively). The off period follows a log-normal distribution with different location parameter for each user, selected with a uniform distribution from [4.068, 6.215], and a scale parameter equal to 0.368. This corresponds to a client demand ($d_{u}$) that varies from 33 to 267 minutes (in total) per epoch. The pareto and log-normal distributions have been used in studies of wireless traffic to describe the session on and off durations \cite{42}.

\textit{Estimation of blocking probability:} For the service selection of users, the blocking probability is estimated

\begin{itemize}
  \item This value was chosen to transform the prices of the subscription and flex services in the same scale as the values of the blocking probability.
\end{itemize}
based on the status of recent sessions reported at u-map during the last 2 epochs.

**User mobility:** During off periods, clients move with a pedestrian speed with a maximum value of 1 m/s, while they remain stationary during sessions. In this work, we do not consider handovers: a client remains connected at the same BS for the entire duration of the session. In general, a session could be blocked not only at its initiation (as in this work) but also during a handover between BSs. The estimation of the blocking probability of handover sessions is an ongoing work.

To highlight the impact of the flex service, two market types were simulated: a subscriber-only market (baseline), in which each user can either become a subscriber of a provider or remain disconnected, and a mixed market, in which users have the additional option of becoming flex users. The analysis evaluates the impact of the flex service on the evolution of the market, using metrics that can provide insights to regulators, users, and providers. The performance of a provider is characterized by its revenue, while the performance of a client is indicated by the blocking probability of its sessions. Furthermore, the session blocking probability, social welfare, market share, and percentage of disconnected users are computed. The session blocking probability of a client is the ratio of its blocked sessions over the total number of session requests. The social welfare is defined as the ratio of the net benefit of all users and providers. The net benefit of a provider is its revenue, while the net benefit of a user is the difference of the user valuation for wireless connectivity and what the user paid for his/her wireless access. The user valuation for wireless connectivity is the price that the user is willing to pay when the blocking probability is zero. Finally, the market share is a vector that shows the percentage of users that choose each service. Our reported results are average statistics over all epochs.

5.2 **Analysis of flex service at the micro level**

To evaluate the performance of the flex service, we performed a series of experiments. The game of users was simulated at the microscopic level, while the game of providers was executed at the macroscopic level to reduce the computational complexity of the price setting algorithm. The flex service improves the performance of users. It significantly reduces the percentage of disconnected users (Fig. 7b), increases the social welfare (Fig. 7c), and improves the blocking probability in most scenarios (Fig. 7a). Provider 1 has an advantage in the revenue over Provider 2 due to the larger channel availability of its BSs. In some cases, the market manifests strong oscillations. The oscillations are caused by the relatively “stale” data in the estimation of the blocking probability that is employed for the service selection of users. The blocking probability is estimated by u-map, based on historical feedback that users upload in u-map about the status of their sessions. Specifically, in certain epochs, the percentage of subscribers of Provider 2 falls close to zero. After some epochs, u-map reports a low blocking probability for the subscribers of Provider 2. This low blocking probability in conjunction with a low subscription rate motivates many users to become subscribers of Provider 2. The massive “flow” of users

![Fig. 7: Performance at the microscopic level: (a) Blocking prob. (b) Disconnected users (%) (c) Social welfare (d), (e) Revenue of providers (f) Market share in the mixed market with a disconnection threshold of -0.175.](image)
to Provider 2 results in an increased blocking probability. Users will realize a performance degradation after some time (due to the uploading delay), and then, they will once again abandon Provider 2. This phenomenon will be repeated (Fig. 7f). The intensity of the oscillations depends on the flex service, market share, uploading frequency, and number of epochs based on which the blocking probabilities of the various services are estimated. The higher the intensity of the oscillations, the higher the average blocking probability. As a result, in certain scenarios, the blocking probability in the mixed market is slightly larger compared to the subscriber-only one (Fig. 7a).

The use of “stale” data causes these oscillations, and the suboptimal performance. Similar oscillations have been also observed in agent economies and biological systems. An extensive discussion can be found in [1]. To improve the estimation of the blocking probability, the Markov-chain model described in Section 4.3 can be employed.

In our earlier study [1], the user utility function was not a weighted sum of the blocking probability and cost. Instead, there were two types of scenarios in which all users were selecting the service that minimizes either the cost or the blocking probability. Part of the user profiles were a willingness-to-pay and a blocking probability tolerance threshold. The benefits of the flex service were also prominent under such user profiling, namely an improvement in the number of disconnected users, blocking probability, and social welfare as well as some trends (e.g., oscillations in market share). The flex service was a preferable choice for users with a low blocking probability tolerance and users with low traffic demand. Regarding providers, that study indicated some interesting phenomena: When clients select BS based on the data rate, the revenue of providers increases in the mixed market compared to the subscriber-only one. The reverse trend holds when clients select BS based on the price. Furthermore, under large user population (e.g., 28000 users), the provider with most resources outperforms in terms of revenue (this was also observed here), while under small population size (e.g., 5000 users), the provider with the best channel quality has the advantage.

5.3 Comparative analysis at micro & macro levels

We performed a series of simulations of the game of users and the game of providers at the macroscopic level. As it happens also at the microscopic level, the flex service improves the performance of users. Specifically, it reduces the percentage of disconnected users (Fig. 8b) and blocking probability (Fig. 8a) in most cases. Furthermore, in the mixed market, the social welfare is similar or slightly larger compared to the subscriber-only one (Fig. 8c). Another common trend of the microscopic and macroscopic levels is that Provider 1 has the advantage in terms of revenue (Figs. 8d and 8e). The impact of the mean value of the user disconnection threshold ($-k$) was also analyzed. At both the microscopic and macroscopic level, the revenue of the providers decreases, when the disconnection threshold of users increases. The higher the disconnection threshold, the more likely
a user will choose to be disconnected. This has as a result low prices and revenue.

Significant differences of the performance of the two levels also exist. The macroscopic-level models underestimate the blocking probability compared to the microscopic-level ones (Figs. 7a, 8a). This is due to the difference in the market share at the two levels. In the mixed market, at the macroscopic level, the user population usually converges to a stable equilibrium point in which there are only flex users (e.g., Fig. 8f). In general, the flex service is characterized by a much lower blocking probability compared to subscriptions. Therefore, at the macroscopic level, the tendency of users to select the flex service results in a reduced average blocking probability. On the other hand, at the microscopic level, the market share exhibits intense oscillations, as explained earlier. These oscillations cause an increased number of users to select a certain service, during some epochs, raising the average blocking probability. Furthermore, the percentage of disconnected users is larger at the microscopic level compared to the macroscopic one (Figs. 7b, 8b). The market oscillations at the microscopic level result in very high blocking probabilities at some epochs. Therefore, in the subsequent epochs, many users choose to become disconnected. Moreover, at the microscopic level, the user traffic demand varies. This means that at the microscopic level, there are users with higher demand than at the macroscopic level. When these users choose the flex service, they may need to pay a high price which sometimes surpasses their tolerance, leading to an increased percentage of disconnected users.

In some cases, providers exhibit different performance depending on the level. Specifically, at the macroscopic level, in most scenarios, the revenue in the mixed market is larger compared to the subscriber-only one (Figs. 8d, 8e). On the contrary, at the microscopic level, the revenue in the mixed market is larger compared to the subscriber-only one when the disconnection threshold is relatively high (e.g., -0.125, -0.100). In the remaining scenarios, the revenue in the mixed market is similar or even smaller compared to the subscriber-only one (Figs. 7d, 7e). This is due to the level that the price setting algorithm simulates the game of users. The estimation of the revenue at the macroscopic level (for the price setting) is not as accurate as the estimation at the microscopic level, leading to suboptimal performance.

As mentioned earlier, at the macroscopic level, in the mixed market, the user population usually converges to an equilibrium, in which there are only flex users. Therefore, the offered subscription rates do not significantly affect the performance of the market. For example, in some scenarios, Provider 2 can not attract subscribers by offering a very low subscription rate. In such scenarios, the market usually converges to an equilibrium in which Provider 2 offers the lowest possible subscription rate. However, users still choose the flex service and the performance of the market is not affected by this strategy of Provider 2. On the contrary, at the microscopic level, conditions are different. The population of users is heterogeneous, possibly with users that prefer subscriptions over the flex service.
For example, a user with high traffic demand may prefer the flat rate of subscriptions. Therefore, by offering a very low subscription rate, Provider 2 may lose profit from these users. Furthermore, the very low subscription rate of Provider 2 intensifies the oscillations. This may result in an increased blocking probability that may surpass even the one of the subscriber-only market.

The above example shows that pricing decisions performed at the macroscopic level may lead to suboptimal performance when used at the microscopic level (even if they are optimal at the macroscopic level), both from the perspective of users and providers. This further motivates the design of the mesoscopic levels. The price setting algorithm could be formulated at a mesoscopic level to achieve a good tradeoff between accuracy and computational complexity. This can prevent providers from taking wrong decisions, and at the same time, keeps the computational complexity of the price setting algorithm relatively low.

### 5.4 Performance at mesoscopic levels

To quantify the tradeoff between accuracy and computational complexity, we simulated the game of users at several levels of detail. Specifically, we defined games with 1, 4, 20, 100, 500, 1000, and 28000 user clusters according to the mesoscopic modeling. The games with 1 and 28000 clusters correspond to the macroscopic and microscopic levels, respectively. In each experiment, the game of providers was simulated at the macroscopic level.

In general, the larger the number of user clusters, the closer the performance of the mesoscopic level to the microscopic level (Figs. 9a - 9e). The microscopic level “corresponds” to the ground truth, since it models each user as a distinct entity, with its complete profile (as opposed to other levels that apply aggregations). The larger the number of user clusters, the more accurate the results. Furthermore, as mentioned in Section 4.3.1, the larger the number of user clusters, the larger the computational complexity. We quantified this trend and measured that the difference in the execution time of the microscopic and macroscopic levels is up to three orders of magnitude! (Fig. 9f).

The macroscopic level tends to overestimate the revenue of providers, especially in the mixed market (Figs. 9d and 9e). Specifically, at the macroscopic level, the estimated revenue in the mixed market is higher compared to the subscriber-only one. However, the reverse trend appears in all other levels. As mentioned, to reduce the computational requirements, the price setting is performed at the macroscopic level. An inaccurate prediction of revenue may mislead a provider in taking decisions that result in suboptimal performance. Instead of simulating the game of providers at the macroscopic level, a mesoscopic level that achieves a good tradeoff between accuracy and computational complexity can be selected (e.g., the mesoscopic level with 100 user clusters). Actually, in experiments in which the simulator of the game of providers was based on this mesoscopic level, we did observe an increased revenue of providers compared to the corresponding case at the macroscopic level (omitted due to lack of space).

### 6 Conclusions and future work

To analyze access markets, we developed a modular multi-layer modeling framework and simulation platform that takes into consideration a diverse set of user profiles and various performance metrics. This paper shows the framework importance and practical merit by investigating the roll out of a new service, the flex service. The analysis demonstrates the following trends: the flex service dramatically reduces the percentage of disconnected users, decreases the blocking probabilities, and improves the social welfare.

For the evaluation of the framework, we comparatively analyzed the markets at the macroscopic and microscopic levels. The two levels manifest similarities: e.g., the flex service improves the performance of the market with respect to the blocking probability and percentage of disconnected users. The analysis also reveals important differences. Interestingly, while at the macroscopic level, the revenue of both providers is larger in the mixed market compared to the subscriber-only one, at the microscopic level, this does not always hold. Furthermore, the market exhibits intense oscillations at the microscopic level, while at the macroscopic level, it usually converges to a stable equilibrium. The competition among providers as well as the delay in the dissemination of the user feedback via u-map cause strong oscillations at the microscopic level. In addition, the difference in the execution time of the microscopic and macroscopic levels is prominent.

The multi-layer modeling pays off. The simulation of a market at multiple levels of detail provides a quantitative measure of the tradeoff between accuracy and complexity and can help us select the appropriate level. Our long-term objective is to assess an extended set of services offered by a number of providers, and various partnerships among providers, including MVNOs and femtocells, in wireless access markets. The present work sets the foundations for a multi-layer modeling of such large-scale wireless access markets.

### References
