Pricing for Mobile Virtual Network Operators: 
The contribution of u-map

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Abstract—The mobile traffic growth is a catalyst of further technological advancements and evolution in wireless access markets. This work focuses on the joint problem of price setting for a Mobile Virtual Network Operator (MVNO), which serves end users in the retail market, and for the Mobile Network Operator, which leases its network to the MVNO at the wholesale market. We show that information about user profiles enables operators to achieve maximum profit. U-map, a crowd-sourcing platform that collects users preferences and builds user profiles, can provide such information. The analysis demonstrates that when only one operator employs (partial) information, the profit of both operators is affected. Furthermore, the cooperation between the operators results in higher profit compared to blind competition.

I. INTRODUCTION

The mobile traffic growth is a catalyst of further technological advancements and evolution in wireless access markets. The global mobile traffic will grow 13-fold from 2012 to 2017, a compound annual growth rate of 66% [1]. At the same time, wireless communications change in a fast pace from the network operators’ and service providers’ perspective [2], [3]. However not only the networks but also the business models of the communication services over these networks change. In particular, the network virtualization [4], a relatively new concept, allows network operators to lease parts of their network. In this context, Mobile Virtual Network Operators (MVNOs) [5] may now find a new vital space to flourish. MVNOs do not own a network but lease access in the wholesale market from a Mobile Network Operator (MNO) and resells access services in the retail market [6].

Typically, MVNOs target specific user populations, services, and regions, such as low-willingness-to-pay users (with low charge-per-month offers), customers on its own network (with free minutes and sms for its customers), youngsters (e.g. with unlimited sms, voice minutes over the same network), web users (e.g. with only data offers), ethnic (special offers for calls to specific destinations) and roaming users (calls, data). MVNOs tend not to compete directly with their host MNO or with MNOs in general. Rather, they aim to widen and deepen the market through brand appeal, targeting of niche markets, and alternative distribution channels. The consumer-welfare impact of MVNOs is in offering extended and innovative services as opposed to lower prices. While some countries have many MVNOs, typically only three in each country have the majority of MVNO customers. Moreover, the combined MVNO market share per country is less than 10% [7].

This work focuses on the economics of MVNOs. In particular, we study the pricing model of MVNO in the retail market and the charging scheme of MNO in the wholesale market. Furthermore, we are particularly interested in assessing the impact of the knowledge about user profile on the profit of MVNOs and MNOs. Operators collect such information mostly from surveys. We have developed u-map, a Quality of Experience (QoE)-based geo-database recommendation tool for selecting the appropriate wireless provider based on user-specific criteria, such as cost and QoE [8], [9]. U-map employs the crowd-sourcing paradigm and collects network measurements and user profile data from mobile users in a geo-database. This profile includes information related to the user willingness-to-pay (WtPay), i.e. how much the user is willing to pay at most for the provided service, the preference of both operators is affected. Furthermore, the cooperation between the operators results in higher profit compared to blind competition.

Several works have also considered the pricing of MVNOs and MNOs. In [10], Guijarro et al. model a wireless access market, where an MVNO leases spectrum from an MNO, which is also active in the retail market, and resales it to end-users. The competition between operators is modeled and the optimal amount of spectrum to be leased is determined. In [11], Debbah et al. study a similar market and introduce the notion of brand appeal. They show that, in the most viable scenario, MNO concentrates on the segment which is sensitive to its brand appeal and let the MVNO access the market. In [12], Cadre et al. tackle the question whether cooperative content investment is profitable for both operators. In [13], [14], Duan et al. propose and study the model of Cognitive MVNO, which has a further source of spectrum, opportunistic access. Contrary to previous approaches that assume perfect knowledge of users preferences, the focus of this work is to quantify the impact of imperfect information.

This paper examines how service providers and network operators can take advantage of u-map to design their charging strategies and improve their profit. In particular, Section II focuses on the discovery of optimal pricing under perfect
knowledge of user profile by both the MVNO and MNO. In Section III, we tackle the issue of partial knowledge and quantify the value of information and the contribution of u-map. Finally, Section IV summarizes the main conclusions and future work plans.

II. PRICING FOR MVNO AND MNO

In this paper, we assume a retail market where an MVNO sells mobile services to a set \( I \) of end users \((i \in I)\). The traffic demand \( d_i \) (in MB/month) of end users are independent and identically distributed random variables with cumulative distribution function (CDF) \( F_d (x) \) and corresponding probability density function (PDF) \( f_d (x) \). WiPay for a certain service of a user is denoted as \( p_{\text{max}} \). WiPays are also modeled as independent and identically distributed random variables with CDF \( F_{p_{\text{max}}} (x) \) and PDF \( f_{p_{\text{max}}} (x) \).

In the retail market, MVNO employs a pricing scheme \( p (\cdot) \) and charges each end user with \( p (d_i) \). From the user’s side, if the user is willing to pay the amount charged with, he/she pays and receives the service; otherwise, he/she abstains. Formally, the user receives service if \( p (d_i) \leq p_{\text{max}} \) or, equivalently, \( \Theta (p_{\text{max}} - p (d_i)) = 1 \), where \( \Theta (x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} \) is the Heaviside step function. The total revenue that MVNO collects and receives the service; otherwise, he/she abstains. Formally, the user is willing to pay the amount charged with, he/she pays \( p_{\text{max}} \).

The total revenue that MVNO collects from the retail market \( R_V (p) \) can be derived as,

\[
R_V (p) = \sum_{i \in I} p (d_i) \Theta (p_{\text{max}} - p (d_i)) \tag{1}
\]

whereas the traffic demand served,

\[
D (p) = \sum_{i \in I} d_i \Theta (p_{\text{max}} - p (d_i)) \tag{2}
\]

Since MVNO does not own a network but leases it from an MNO in the wholesale market, MNO charges MVNO with \( c (D) \) and its total profit is,

\[
P_N (p, c) = c \left( \sum_{i \in I} d_i \Theta (p_{\text{max}} - p (d_i)) \right) = \sum_{i \in I} c (d_i) \Theta (p_{\text{max}} - p (d_i)) \tag{3}
\]

where \( c \) is a linear function [6]. Consequently, the profit for MVNO \( P_V (p, c) \) can be derived as,

\[
P_V (p, c) = R_V (p) - P_N (p, c) = \sum_{i \in I} (p (d_i) - c (d_i)) \Theta (p_{\text{max}} - p (d_i)) \tag{4}
\]

This work aims to study the share of wealth that MVNO and MNO can earn from the market at the infinity regime, where the number of users in the market tends to infinity \((I = |I| \rightarrow +\infty)\). The total wealth in the market is \( W = \sum_{i \in I} p_{\text{max}} \) and the wealth share for MVNO and MNO at the infinity regime can be derived as,

\[
P_V \xrightarrow{I \rightarrow +\infty} \frac{E ((p - c) \Theta (p_{\text{max}} - p))}{E (p_{\text{max}})} = S_V^\infty (p, c) \tag{5}
\]

\[
P_N \xrightarrow{I \rightarrow +\infty} \frac{E (c \Theta (p_{\text{max}} - p))}{E (p_{\text{max}})} = S_N^\infty (p, c) \tag{6}
\]

where, for brevity, \( p = p (d) \) and \( c = c (d) \) and the expectation \( E \) is over all the involved random variables. The analytical calculation of the wealth share for MVNO and MNO is given in (7) and (8), respectively, where \( G_{p_{\text{max}}} (p, c) = (p - c) \frac{1 - F_{p_{\text{max}}} (p)}{E (p_{\text{max}})} \) and \( K_{p_{\text{max}}} (p, c) = c \frac{1 - F_{p_{\text{max}}} (p)}{E (p_{\text{max}})} \).

MVNO aims to maximize its wealth share given the wholesale price. To achieve this, it is sufficient to maximize \( G_{p_{\text{max}}} (p, c) \) with respect to \( p \). Formally, if \( p^* \) is the optimal price scheme that maximizes the wealth share of MVNO, it holds that,

\[
[1 - G_{p_{\text{max}}} (p^*)] - (p^* - c) G_{p_{\text{max}}} (p^*) = 0 \tag{9}
\]

\[
2 f_{p_{\text{max}}} (p^*) + (p^* - c) \frac{d f_{p_{\text{max}}} (p^*)}{d p} \bigg|_{p=p^*} > 0 \tag{10}
\]

Similarly, MNO also tries to maximize its wealth share by means of maximizing \( K_{p_{\text{max}}} (p, c) \) with respect to \( c \). Contrary to MVNO, which cannot control the wholesale charge \( c \) (considered constant), MNO can affect the retail price \( p \), which is now considered as a function of \( c \). Consequently, if \( c^* \) is the wholesale charging scheme that maximizes MNO’s wealth share, it holds that,

\[
[1 - K_{p_{\text{max}}} (p (c^*))] - c^* K_{p_{\text{max}}} (p (c^*)) \frac{dp}{dc} \bigg|_{c=c^*} = 0 \tag{11}
\]

\[
2 f_{p_{\text{max}}} (p (c^*)) \left( c^* \frac{d f_{p_{\text{max}}} (p (c^*))}{d c} \right) \bigg|_{c=c^*} > 0 \tag{12}
\]

Assuming that MVNO has selected its optimal pricing as of (9), then from (9) and (11), it follows that

\[
p^* (c^*) - c^* \left( 1 + \left| \frac{dp}{dc} \right| \bigg|_{c=c^*} \right) = 0 \tag{13}
\]

From the previous analysis, it is clear that if both operators aim at maximizing their wealth share, their pricing policies at the wholesale and the retail market depend only on the distribution of users’ WtPay. This conclusion is rather expected since, in the absence of any limitation on the capacity of the network, the only constraint imposed is users’ WtPay. The operators tune their pricing schemes so as to reap the maximum wealth share independently from the traffic generated by each user. To a certain extent, the assumption of unlimited capacity can be supported by the constant growth of the networks and the technological advancements in this field that tend to provide a “virtually” unlimited capacity. This conclusion is also in accordance with the latest trend of unlimited allowance tariff plans in the mobile communications market.

To gain further insight on the pricing behavior of both operators, we also examine the specific one, where the WtPay follows exponential distribution with mean value \( \mu \) [10]. Then, \( F_{p_{\text{max}}} (x) = 1 - e^{-\frac{x}{\mu}} \) and \( f_{p_{\text{max}}} (x) = e^{-\frac{x}{\mu}} / \mu \). From (9), we derive the optimal price scheme for MVNO as \( p^* = \mu + c \), which also satisfies (10). As a result, the pricing strategy of MVNO is to transfer the wholesale cost directly to the
Furthermore, users preferences change dynamically in a fast rate so as to earn profit. For MNO to maximize its wealth share, (13) must hold, which results that \( c^* = \mu \), which also satisfies (12), thus \( p^* (c^*) = 2\mu \). From (7) and (8), the maximum wealth share for MVNO and MNO is calculated as 
\[
S_N^\infty (p^*, c^*) = S_N^\infty (p^*, c^*) = e^{-2}.
\]
As a result, MVNO and MNO equally share the profits reaped from the market.

Additionally to the previous analysis, we have simulated a wireless access market with \( I = 10^7 \) users, whose WtPay follows an exponential distribution with mean value \( \mu = 1 \). MVNO tunes its price \( p \) as MNO’s price \( c \) varies. Hence, MNO selects the optimal wholesale price \( c^* \) given MVNO’s pricing strategy \( p^* (c) \). Fig. 1a depicts the price \( p^* \) for MVNO that maximizes its wealth share vs. the wholesale charge \( c \) of MNO. The wealth share for MVNO and MNO when MVNO employs its optimal price \( p^* \) vs. the wholesale charge \( c \) of MNO are presented in Fig. 1b. The simulation curves closely follow the analytical ones. Furthermore, the optimal wholesale charge is \( e^* = \mu = 1 \), which leads to 
\[
S_N^\infty (p^*, c^*) = S_N^\infty (p^*, c^*) = e^{-2} \approx 13.53\% \quad \text{(curve } S_N^\infty \text{ in Fig. 1b)}.
\]

### III. The Value of Information

The previous section focuses on the price setting for MVNO and MNO under perfect knowledge of users profile. However, it is unrealistic to assume the availability of perfect knowledge about user profiles at MVNOs and MNOs. The partial knowledge collected based on current methods, such as surveys, may be inaccurate, biased, partial, and certainly costly. Furthermore, users preferences change dynamically in a fast rate so as to earn profit. For MNO to maximize its wealth share, (13) must hold, which results that \( c^* = \mu \), which also satisfies (12), thus \( p^* (c^*) = 2\mu \). From (7) and (8), the maximum wealth share for MVNO and MNO is calculated as 
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MVNO and MNO gather information from a number of users in the market and estimate the parameters of the WtPay distribution. Due to space limitations, we assume that the WtPay distribution parameter estimation is accurate. As a result, the loss that each one suffers (\( L_N^\infty \), \( L_N^\infty \)) from this inaccurate estimation is as follows,
\[
L_N^\infty = 1 - \frac{S_N^\infty (\tilde{p}^*, \tilde{c}^*)}{S_N^\infty (p^*, c^*)} \quad \text{and} \quad L_N^\infty = 1 - \frac{\tilde{c}^*}{\tilde{c}^*} \quad \text{whereas MVNO decides to charge end users with } \tilde{p}^* = \tilde{\mu} + \tilde{\mu} \text{. As a result, the loss that each one suffers (\( L_N^\infty \), \( L_N^\infty \)) from this inaccurate estimation is as follows,}
\]
\[
L_N^\infty = 1 - \frac{S_N^\infty (\tilde{p}^*, \tilde{c}^*)}{S_N^\infty (p^*, c^*)} \quad \text{and} \quad L_N^\infty = 1 - \frac{\tilde{c}^*}{\tilde{c}^*} \quad \text{whereas MVNO decides to charge end users with } \tilde{p}^* = \tilde{\mu} + \tilde{\mu} \text{. As a result, the loss that each one suffers (\( L_N^\infty \), \( L_N^\infty \)) from this inaccurate estimation is as follows,}
\]
\[
\begin{align*}
L_N^\infty & = 1 - \frac{S_N^\infty (\tilde{p}^*, \tilde{c}^*)}{S_N^\infty (p^*, c^*)} = 1 - \frac{\tilde{\mu} e^{-2(\tilde{\mu} + \tilde{\mu})}}{	ilde{\mu}} \\
L_N^\infty & = 1 - \frac{\tilde{c}^*}{\tilde{c}^*} = 1 - \frac{\tilde{\mu} e^{-2(\tilde{\mu} + \tilde{\mu})}}{\tilde{\mu}}
\end{align*}
\]

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![Fig. 1. (a) Optimal MVNO price \( p^*(c) \) and (b) wealth share for MVNO \( S_N^\infty (p^*(c), c) \) and MNO \( S_N^\infty (p^*(c), c) \) at optimal MVNO price \( p^*(c) \) vs. MNO price \( c \).](image1)

![Fig. 2. (a) MVNO loss \( L_N^\infty (Th) \) and (b) MNO loss \( L_N^\infty (Th) \) vs. number of users for WtPay distribution parameter estimation.](image2)
The loss for the two operators is depicted in Fig. 2a and 2b. Interestingly, the loss can be positive or negative, i.e., there is loss as well as gain from the inaccurate estimation of $\mu$. In particular, if both operators overestimate the willingness of end users to pay, i.e., $\hat{\mu} > \mu$ and $\hat{\mu} > \mu$, they increase the retail and wholesale prices, driving thus more users to abstention from market. This results in loss for both operators ($L_N^\infty > 0$, $L_N^\infty > 0$). If one operator overestimates the WtPay whereas the other underestimates it, i.e., $\hat{\mu} > \mu$ and $\hat{\mu} < \mu$ or $\hat{\mu} < \mu$ and $\hat{\mu} > \mu$, the operator that underestimates WtPay charges less than optimal, so the loss is unavoidable. However, the fact that this operator charges less than optimal widens the profit gain for the other operator. In other words, the other operator can now charge more. That is why the other operator has gain from the overestimation of WtPay, because it charges more than optimal. Of course, overestimation is useful up to the point that $\hat{\mu} < -W_{-1}(-e^{-2}) \mu$ or $\hat{\mu} < -W_{-1}(-e^{-2}) \mu$ for MVNO and MNO, respectively, where $W_k(\cdot)$ is the $k$-th branch of the Lambert W function ($W_{-1}(-e^{-2}) \approx -3.15$). From this point onward, the operator has consumed the advantage gained and loss follows.

The most interesting case is when both operators underestimate the WtPay, i.e., $\hat{\mu} < \mu$ and $\hat{\mu} < \mu$. In the intersection of the zero-loss curves, both operators can have a gain (Figs. 2a and 2b). When $\hat{\mu} = \mu = \mu/2$ both operators gain $1 - e/2$ and their wealth share becomes $S_N^\infty(\hat{p}^*, \hat{c}^*) = S_N^\infty(\hat{p}^*, \hat{c}^*) = e^{-1}/2 \approx 18.39\%$. We observe that the competition does not allow the operators to explore a better operational point for both. By considering the cooperation, instead of the blind competition, they could have increased their wealth share by 35.91% approximately (4.86% absolute increment). Finally, if the underestimation of WtPay falls below $-W_0(-e^{-2}) \mu$ ($W_0(-e^{-2}) \approx -0.16$), both operators lower their prices that much that they cannot recover the loss (due to low price) from the increased user participation and loss again incurs.

We estimate the loss of MVNO and MNO as a function of the number of users employed for the estimation of the parameter of the WtPay distribution (Figs. 2c & 2d, respectively). This experiment assumes $I = 10^7$ number of users in the market and $\mu = 1$. For each user population, we have conducted 100 estimations of $\mu$ and present the maximum, mean, and minimum loss encountered by the operators for this sample size. These results confirm the findings of the previous analysis: the loss can be positive or negative for both operators. Although the mean value converges very fast to zero, more than 200 users profiles are required to ensure the maximum loss remains below 20%. U-map offers a timely and cost-effective way to collect this information.

IV. CONCLUSIONS AND FUTURE WORK

This work studies a wireless access market, where an MVNO serves end users in the retail market and an MNO provides access to the MVNO in the wholesale market. The pricing policies for both operators that maximize wealth share were found. We have shown that, in the absence of a capacity constraint, the optimal pricing strategies for both operators depend only on the distribution of users’ WtPay. The contribution of u-map in collecting information about users preferences and building users profiles has been highlighted. The impact of the availability of such information (e.g., traffic and willingness to pay) on the design of tariff plans was analyzed. We demonstrated that partial information may cause profit loss, but, due to the interaction between the operators, it is also possible to reap gain.

We plan to study how the prices evolve when there is competition in the retail market, under the presence of more than one operators, and especially, when the MNO is also active in the retail market. The competition in the wholesale market as well as the cooperation between MVNO and MNO is also of interest. Finally we plan to model and analyze the tradeoff of QoE vs. price and its impact on price setting by considering cases where the larger the traffic of an operator, the poorer the QoE.

REFERENCES


