Belief Change in Arbitrary Logics

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ABSTRACT
The problem of belief change refers to the updating of a Knowledge Base (KB) in the face of new, possibly contradictory information. The AGM theory, introduced in [1], is the dominating paradigm in the area of belief change. Unfortunately, the assumptions made by the authors of [1] in the formulation of their theory restrict its use to a specific class of logics. In this work, we investigate the possibility of extending their framework in a wider class of formalisms by examining the applicability of the AGM postulates in logics originally excluded from the AGM model. We conclude that in the wider class that we consider there are logics which do not admit AGM-compliant operators. As a case study, we investigate the applicability of the AGM theory to Description Logics (DLs). Furthermore, we use our results to shed light on our inability to develop AGM-compliant operators for belief bases.

Categories and Subject Descriptors
H.2.3 [**Languages**]: Data Manipulation Languages

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Languages, Theory

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1. INTRODUCTION – PREVIOUS WORK
One of the crucial actions any Knowledge Representation (KR) system must undertake is the updating of its Knowledge Base (KB) in the face of new information that is possibly contradictory with its current beliefs. This problem is usually referred to as the problem of *belief change*. Being able to dynamically change the stored data in a KB is very important for several reasons; mistakes may have occurred during the input; or some new information may have become available; or the world represented by the KB may have changed. In all such cases, the stored beliefs should change to reflect this fact.

The most influential work on belief change is [1], where three different types of belief change were considered, namely *expansion*, *revision* and *contraction*. Expansion is the addition of a sentence to a KB, without taking any special provisions for maintaining consistency; revision is similar, with the important difference that the result should be a consistent set of beliefs; contraction is required when one wishes to consistently *remove* a sentence from their beliefs instead of adding one.

Instead of trying to find a specific method of dealing with these belief change operators, the authors of [1] chose to investigate the properties that each such method should have in order to be intuitively appealing. The result was a set of postulates (named *AGM postulates* after the initials of the authors) that every belief change operator should satisfy. This paper had a major influence in most subsequent works on belief change, being the dominating paradigm in the area ever since.

Given the (almost) universal acceptance of the AGM postulates as the defining paradigm for belief change operations, it would be desirable to apply them in any type of logic. Unfortunately, the original AGM theory contains some non-elementary assumptions about the logic at hand which disallow its direct application to some classes of logics, such as Description Logics (DLs) and equational logic.

To our knowledge, there has been no study on the possibility of applying the AGM paradigm in such logics. This work attempts to fill this gap, by extending the AGM approach to a wider class of logics, containing several interesting logics originally excluded from the AGM model. We apply our results to study the relationship of the AGM postulates with DLs, one of the leading formalisms for storing and manipulating knowledge in the Semantic Web. We also study belief change operations on belief bases, a set of operations based on slightly different assumptions than the AGM model, as well as their relationship with the standard AGM paradigm. Initially, we deal with the operation of contraction only; dealing with revision and other non-trivial belief change operators such as revision and even update and erasure (see [7] for the definition of these operators) is part of our future work. In this text, we will briefly and informally present the most important initial results of our work; a more complete and formal presentation can be found in [3] and [4].

2. SETTING AND TERMINOLOGY
In this text, the term *logic* will refer to a pair $<L, Cn>$, where L is a set of expressions and Cn is a function (consequence operator) mapping subsets of L to subsets of L. The set L contains all the available propositions of the logic; for example, in propositional calculus, $a \wedge b$, $\neg a$, $a \vee (\neg b) \in L$. The consequence operator returns all the implications of a given set of sentences; we assume that

this operator satisfies the *Tarskian axioms*, namely *iteration* (Cn(Cn(A))=Cn(A)), *inclusion* (A⊆Cn(A)) and *monotony* (A⊆B implies Cn(A)⊆Cn(B)). We will say that A *implies* B iff B⊆Cn(A).

Notice that, unlike the assumptions made in [1], we do not assume the existence of any operators in the logic; this means that we can only "connect" propositions by grouping them in a set. Moreover, the consequence operator is not required to include classical tautological implication, is not necessarily compact and could violate the "rule of introduction of disjunctions in the premises". The above assumptions are general enough to include most interesting classes of logics.

3. CONTRIBUTION

3.1 General Results

One of the results that the AGM proved in [1] is that in any logic (in the class they considered) there is a whole family of contraction operators that satisfy their postulates. It was soon made clear that, once the restrictions of the AGM model are dropped, this is no longer true. In effect, there are logics in the wider class that we consider for which no contraction operator satisfying the AGM postulates (termed *AGM-compliant operator* from now on) can be defined.

A property closely related to the existence of an AGM-compliant operator is *decomposability*. A *decomposable* logic <L, Cn> has the property that for every pair of sets of propositions in L (A, B⊆L) for which it holds that Cn(∅)⊂Cn(B)⊂Cn(A), there exists a third set C (C⊆L) such that Cn(C)⊂Cn(A) and Cn(B∪C)=Cn(A). Informally, every set B implied by A must have a "pair", i.e. another set C implied by A such that B and C together imply A (Cn(B∪C)=Cn(A)). Notice that each of B and C *alone* are proper implications of A, while B and C *together* are equivalent to A. Our study has proven that a logic accepts an AGM-compliant operator iff it is decomposable.

Starting from decomposability, another equivalent formulation of AGM-compliant logics can be developed. A *cut* of a set A is a family S of sets implied by A such that any other set implied by A either implies or is implied by a set in S. It can be shown that the intersection of all sets in a cut in an AGM-compliant logic contains only tautologies. Conversely, if the logic is not AGM-compliant, then there exists a cut which violates the above property, i.e. the intersection of all sets in the cut contains non-tautological propositions as well.

3.2 Description Logics

One important example of application of belief change that cannot be directly accommodated by the AGM approach is the problem of belief change in DLs, an important formalism that is used in several applications in the Semantic Web. The family of DLs is a number of schemes used to represent knowledge. Each type of DL contains a *namespace* for naming its *concepts* and *roles* (the building blocks of a DL KB), a set of *constants* (such as ⊤, ⊥), a set of *operators* (such as ⊓, ⊔, ¬, ∃, ∀, ≥n, ≤n etc) for combining constants, concepts and roles in more complex expressions and a set of *relations* (such as ≅, ⊑ etc) for relating such expressions. For a detailed description of DLs see [2].

One important problem that has been generally disregarded in the DL literature is the problem of updating the Tbox (schema) of a DL KB. It would be useful to find an update operator for DL Tboxes that complies with the AGM postulates. Unfortunately, the AGM theory is inapplicable here. The facts in a DL Tbox are usually of equational nature (such as A≅B⊓C or A⊑(¬B)⊔C); for such facts x one cannot always express ¬x for example; furthermore, many DLs are not compact. Thus, we cannot be sure in advance that an AGM-compliant operator actually exists for this problem. On the other hand, DLs can be easily accommodated within our framework, as shown in [3]; so the results of our work can be directly applied to the problem of DL Tbox updating. An interesting question would be to determine which DLs are AGM-compliant.

To answer this question it would be useful to initially acknowledge two important characteristics of the problem. Firstly, in the DL context, it is usually desirable to assume that concepts and roles that appear nowhere in a DL KB do not exist, as far as the KB is concerned. This assumption is usually made implicitly in the literature. Secondly, in a DL KB we would like to be able to make contractions of the form: "remove all occurrences of the concept/role X from the KB". Such contractions are commonly used in the DL context. These two features are usually taken or dropped together, as they are too closely related and have to do with the semantics given to the concepts and roles.

As we proved in [3], [4], if we accept these two features, we end up in DLs that are not AGM-compliant. In fact, all non-trivial DLs that have these characteristics are not AGM-compliant. However, acknowledging the fact that these features are merely desirable, but not essential for a DL KB, we did not give up on our search for an AGM-compliant DL; instead, we decided to drop these features and search for an AGM-compliant DL that does not support them. Unfortunately, our research has not revealed such a DL yet; on the contrary, we proved that most DLs of the AL family (more precisely the DLs: FL_0, FL^-, AL and all DLs of the family AL[U][E][N][C]) are not AGM-compliant (see [2] for the definition of these DLs). Despite this negative result, it is possible that some of the more (or less) expressive DLs are AGM-compliant. Thus, the exact connection between DLs and AGM-compliance is an interesting topic of undergoing work.

3.3 Base Contraction Operators

One of the criticisms the AGM model had to face was the fact that there is no discrimination between explicit facts (obtained directly from observations, measurements, experiments or certain rules regarding the domain) and implicit facts, which are merely implied by the explicit ones. This was considered counter-intuitive (see [5], [6], [8] for some arguments in favor of this opinion). Furthermore, theories are infinite structures, so the development of an algorithm that deals with them is rather problematic.

The proposed alternative was to deal with belief bases, which are sets of propositions not necessarily closed under logical consequence. Such sets contain only the explicit facts. This approach has a severe effect on the contraction operators considered; when contracting a fact from a KB, one should remove only explicit facts to accommodate the contraction. Furthermore, one cannot add any other facts in the base, not even those implied by the facts in the original KB. This limits our options for a contraction operator, because, in the standard AGM model, one can remove explicit and/or implicit facts without discrimination.

In [3], [4] and [5] the counterparts of the AGM postulates for belief base contraction operations were presented. It is easy to see that most logics, even those under the original AGM framework, do not admit operators that comply with these postulates. Due to this observation the AGM theory was characterized unsuitable to describe base contraction operators by most researchers.

Our study showed that the base contraction postulates are not themselves inconsistent; instead there are certain conditions that allow a logic to admit a *base-AGM-compliant* operator, i.e. an operator that satisfies the AGM postulates for base contraction. These conditions are so strong that many logics, including propositional calculus and first-order logic, fail to satisfy them. It was also showed that there are very close connections between AGM-compliance and base-AGM-compliance; these connections allowed us to develop counterparts of most theorems regarding AGM-compliance to refer to base-AGM-compliance.

4. CONCLUSION AND FUTURE WORK

We studied the applicability of the AGM postulates in a class of logics wider than the one originally considered in the AGM model. The original AGM model fails to accommodate several interesting classes of logics which are included in our framework. We concluded that the AGM postulates can be applied to many such logics; unfortunately though, some of the logics in our wider class do not admit AGM-compliant operators.

We applied our results to prove several types of DLs to be non-AGM-compliant, motivating a search for alternative rationality postulates for updating such DLs; finding an AGM-compliant DL is a topic of ongoing research. Furthermore, we studied belief change operations upon belief bases, their respective postulates and their relation with standard AGM-compliance. We found very close similarities, which are presented in [3].

Further applications of our method include the definition of a notion of equivalence between logics with the property of preserving AGM-compliance. Using this equivalence relation, we can show that the set of logics in our framework is isomorphic to the set of complete lattices (up to equivalence). Another important notion is the notion of *roots* of a logic, referring to expressions which have no proper implications other than tautologies. Such expressions have some important properties, closely related to our theory. Finally, the notion of *max-cuts*, a special type of cuts, allows us to check only one cut to determine decomposability, a property that might allow the development of an algorithm for checking AGM-compliance, at least in the finite case. All these results and applications are presented in detail in [3] and [4].

Several research paths are opened using the above initial results. First of all, we could develop other equivalent formulations of AGM-compliance in the general case or in some special cases. Some open questions regarding the AGM-compliance of several DLs remain; we hope to provide a definite answer to such questions in the near future. Other formalisms, such as equational logic, originally disregarded by the AGM paradigm could also be studied.

Further study on the application of the AGM postulates in belief base operations is necessary to establish the exact connections between the two families of belief change operations. The existence of a base-AGM-compliant logic that is useful in practice looks rather doubtful; it would be interesting if we could find such a logic or prove that it does not exist.

Some types of operations were ignored in our initial study of the problem. Revision, update and erasure are fundamental operations each with its own set of postulates; it is possible that decomposability is connected with these postulates as well. Furthermore, AGM proposed a set of additional postulates for contraction, called *supplementary*; it looks like decomposability is a necessary and sufficient condition for the supplementary postulates to hold as well, but this fact has not yet been proved.

Hopefully, addressing the above issues will provide a clearer picture on the properties of AGM-compliant systems. Such systems are of particular interest because they allow rational belief change operations to be defined, where the word “rational” in this context means “satisfying the AGM postulates”. Our ultimate goal is to characterize some of the systems used today with respect to AGM-compliance (and base-AGM-compliance) and provide a theoretical toolkit that will be useful in relevant future studies. Finally, our work will hopefully provide an additional insight on the ongoing debate regarding the rationality of AGM-compliant operators, especially in the context of belief base operations.

5. REFERENCES

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